

SAT Math Shortcut Questions

What is the sum of the solutions of this quadratic equation?

- The sum of the solutions to ANY quadratic equation in standard form is $-\frac{b}{a}$

What is ___ if the system of equations has no solutions/infinite solutions?

- If a system of equations has no solutions, use $\frac{a_1}{a_2} = \frac{b_1}{b_2}$.
- If a system of equations has infinite solutions, use $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

If two expressions are always equal/equal for all values of x

- (e.g., $ax^2 + bx + c = 4x^2 + 7x - 2$), then each of their parts is equal (e.g. $a = 4, b = 7, c = -2$).

Which of the following is equivalent to a rational expression (like $\frac{5x+2}{x-5}$)?

- Use long division or Making Up Numbers.

For questions about the number or type of solutions to a quadratic equation...

- Evaluate the discriminant (or graph with your calculator).

What is the minimum/maximum...

- Generally, the minimum is the next integer *up*, and the maximum is the next integer *down*.*

Shortcutting percent questions:

- Whatever you're comparing to (look for *than, of, compared to, etc.*) is on the bottom of the fraction. Percent increase must be > 1 , and percent decrease must be < 1 .

In a formula, if something happens to one variable, what happens to another?

- If you can't do it analytically: get rid of numbers and variables that aren't involved. Make Up Numbers for the ones that are involved. Compare two results.

If two angles add up to 90° ...

- The sine of one is the cosine of the other (and vice-versa); same thing with tangent and cotangent. Look for x° and $(90 - x)^\circ$.

If you're asked to factor/find solutions for a cubic (or higher)...

- You can factor by grouping if there are no other quick ways to do it.

Sum of Solutions

1. What is the sum of the solutions to the equation $x^2 + 7x - 3 = 0$?

2. What is the sum of the solutions to the equation $3x^2 - 4x = 7$?

3. What is the sum of the solutions to the equation $5x^2 - 10x - 25 = 0$?

4. What is the sum of the solutions to the equation $11x^2 - 5.5x = 1$?

5. What is the sum of the solutions to the equation $2x^2 = 8x + 1$?

6. What is the sum of the solutions to the equation $3x - 5 = 4x^2 - 6$?

7. What is b if the sum of the solutions to the equation $x^2 + bx + c = 0$ is 5?

8. What is b if the sum of the solutions to the equation $4x^2 + bx + c = 0$ is $-\frac{2}{3}$?

9. What is the sum of the solutions to the equation $(2x - 1)^2 = (x + 3)^2$?

10. What is the sum of the solutions to the equation $(x - 2)^2 = 4x + 1$?

No/Infinite Solutions

$$\begin{aligned}3x - jy &= 7 \\ kx - 4y &= 14\end{aligned}$$

Ex. A: If the system of equations shown above has an infinite number of solutions, what is the value of $j + k$?

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \qquad \frac{3}{k} = \frac{-j}{-4} = \frac{7}{14}$$

Cross-multiply in pairs to get $j = 2$, $k = 6$, and $j + k = 8$

$$\begin{aligned}3x + 4.5y &= c \\ bx + 3y &= -7\end{aligned}$$

1. If the system of equations shown above has an infinite number of solutions, what is the value of $b + c$?

$$\begin{aligned}4.2x - 6y &= 3.9 \\ 7x - my &= c\end{aligned}$$

2. If the system of equations shown above has no solution, what CANNOT be the value of c ?

$$\begin{aligned}ax + by &= 7 \\ 4x - 3y &= c\end{aligned}$$

3. If the system of equations shown above has no solution, what is the ratio of a to b ?

$$\begin{aligned}ax - y &= c \\ 2y - 7x &= b\end{aligned}$$

Ex. B: If the system of equations shown above has no solution, what is a ?

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \qquad \frac{a}{2} = \frac{-1}{-7}$$

Cross-multiply to get $a = \frac{2}{7}$

$$\begin{aligned}ax + by &= 12 \\ 7x - 3y &= 8\end{aligned}$$

4. If the system of equations shown above has infinite solutions, what is the value of $a + b$?

$$\begin{aligned}ax - 4y &= 4 \\ ay - 12x &= 5\end{aligned}$$

5. If the system of equations shown above has no solution and $a > 0$, what is a ?

$$\begin{aligned}2x - 3y &= c \\ 5y - kx &= 25\end{aligned}$$

6. If the system of equations shown above has infinite solutions, what are the values of c and k ?

Equivalent Expressions

If two expressions are always equal, or equal for all values of x , or one can be rewritten as the other, or are equivalent, etc. — then all of their parts are identical.

1. If $(x - 1)(x + 4) = ax^2 + bx + c$ for all values of x , what is b ?

4. If the expression $(2x + a)(2x - b)$ is equivalent to $ax^2 - c$ for all values of x , what is the value of c ?

2. If $(x + j)(x + k)$ is equivalent to $x^2 + 9x - 20$ for all values of x , what is $j + k$?

5. If $(ax + 5)(bx - 2)$ can be rewritten as $12x^2 + cx - 10$, and $a + b = 7$, what are the two possible values of c ?

3. If the expression $x(x + 1) + 4(x + 2)$ is rewritten as $x^2 + bx + 8$, what is the value of b ?

6. If $(x + a)(3x + b)$ is equivalent to $3x^2 - 17x + c$ for all values of x , and $a + b = -9$, what is c ?

Dividing Rational Expressions

Divide the following:

1.

$$\frac{6x + 4}{2x - 1}$$

4.

$$\frac{8x - 2}{2x + 3}$$

2.

$$\frac{7x + 3}{x - 1}$$

5.

$$\frac{3x + 4}{3x - 1}$$

3.

$$\frac{5x + 1}{x - 7}$$

6.

$$\frac{12x + 4}{3x + 4}$$

Factoring By Grouping

1. Factor completely: $x^3 + 7x^2 - 2x - 14$

3. Find all of the zeroes of the function
 $f(x) = x^3 + 4x^2 - x - 4$.

2. Factor completely: $2x^3 - 6x^2 - 3x + 9$

4. Find all of the zeroes of the function
 $g(x) = 3x^3 - x^2 - 75x + 25$.

Evaluating the Discriminant

1. Describe the zeroes of $y = x^2 - 7x + 3$ and its behavior relative to the x -axis.

4. What is c if the equation $y = x^2 + 5x + c$ has exactly one zero?

2. Describe the roots and graph of $y = 2x^2 - 4x + 3$.

5. What is the minimum possible integer value of a given that the function $f(x) = ax^2 + 5x + 3$ does not have any real zeroes?

3. Describe the roots and graph of $y = x^2 + 8x + 16$.

6. What is the largest possible integer value of c given that the graph of the function $g(x) = 2x^2 - 7x + c$ has two distinct x -intercepts?

Maximum/Minimum

1. The largest exterior angle in a polygon is 25° . What is the minimum number of sides this polygon could have?

2. A salesman's monthly pay includes a \$1,000 base salary and a bonus of \$525 for each car he sells. In order to earn at least \$4,000 in a month, what is the minimum number of cars he must sell?

3. The product of two positive numbers, x and y , is 120. If x is greater than 7, what is the maximum possible integer value of y ?

4. Martin is purchasing tiles to cover his kitchen floor. His kitchen is 13 feet by 9 feet. Each tile is 1 square foot and tiles come in boxes of 12. How many boxes of tiles must he purchase?

5. A charity raffle guarantees that at least 6% of the tickets sold will earn a prize. If the charity expects to sell 205 tickets, what is the minimum number of prizes it should expect to give out?

6. A business has an advertising budget of \$17,000. It plans to spend \$3,700 to create a radio advertisement. How many times can it afford to play the advertisement on the radio at a cost of \$450 per play?

7. A plumber's employer pays him \$43 per hour or any portion of an hour that he works. What is the minimum number of whole hours the plumber must work to make \$300 in a day?

8. A product that normally sells for \$119 is discounted by $x\%$. If x is an integer, what is the largest possible value of x for which the resulting price is still more than \$70?

9. A woman is in an arcade which closes at 10:00PM, playing a video game which takes $3\frac{1}{2}$ minutes to play. She knows that she is allowed to finish any video game that she starts before 10:00PM. If she arrived at 9:20PM, what is the maximum number of games she can complete?

Sin/Cos and Tan/Cot

1. If $\sin x^\circ = .7$, what is $\cos(90 - x)^\circ$?
2. If $\cos x^\circ = .35$, what is $\sin(90 - x)^\circ$?
3. If $\sin 22^\circ = \cos x^\circ$ and $0 \leq x \leq 90$, what is the value of x ?
4. If $\cos 84^\circ = \sin x^\circ$ and $0 \leq x \leq 90$, what is the value of x ?
5. If $\sin m^\circ = \cos j^\circ$ and both m and j are between 0 and 90, what is the value of $m + j$?
6. If $y = \sin b^\circ$, $y = \cos a^\circ$, and a and b are both positive, what is the least possible value of $a + b$?
7. If $\tan x^\circ = 1.5$, what is $\cot(90 - x)^\circ$?
8. If $\cot x^\circ = .93$, what is $\tan(90 - x)^\circ$?
9. If $\tan 30^\circ = \cot x^\circ$ and $0 \leq x \leq 90$, what is the value of x ?
10. If $\tan 75^\circ = \cot x^\circ$ and $0 \leq x \leq 90$, what is the value of x ?
11. If $y = \cot b^\circ$, $y = \tan a^\circ$, and a and b are both positive, what is the least possible value of $a + b$?
12. If $c + d = 90$, what is the value of $\cos c^\circ - \sin d^\circ$?

Percent Change

1. What percent more than 43 is 64?
2. 13.3 is what percent more than 1.9?
3. 8 is what percent less than 17?
4. What percent less than 77 is 61?
5. If the price of a stock decreased from \$21.64 to \$20.15, by what percent did the price decrease?
6. If the population of a town is currently 17,000, by what percent must it increase to reach 20,000?
7. A city received 32 inches of rain this year, which was 17% less than it received last year. How much did it receive last year?
8. The regional population of a certain animal must rise by 56% to reach 1,100, a level that is considered healthy for the region. Based on these numbers, what is the current population of this animal?
9. A magazine aims to have 150 pages of paid ads in each issue. If the March issue had 137 pages of paid ads, by what percent did the magazine fall short of its goal?
10. After 2 hours of use, a certain cell phone has 77% of its battery charge remaining. A new type of battery promises a 20% longer battery life. After the same 2 hours of use, what percent of the new battery's charge would remain?

Changing Parts of a Formula

The distance in feet, d , that an object falls in t seconds can be calculated using the formula $d = \frac{1}{2}at^2$, where a is the acceleration due to gravity.

1. If an object is given twice the time to fall, by what factor is the distance of its fall increased?

2. How many times longer does one object need in order to fall twice the distance of another object?

In chemistry, properties of gases in containers are related using the formula $PV = nRT$, where P is pressure, V is the volume of the container, n is the number of moles of the gas, R is a constant, and T is the temperature in Kelvins.

3. If the number of moles of gas is tripled, what happens to the pressure of the gas?

4. If a scientist wants to use a container that is $\frac{1}{3}$ the size and have half of the pressure of his current setup, what fraction of the current temperature should the new temperature be?

The net force required for an object to move in a circle is given by $F_{net} = \frac{mv^2}{R}$, where m is the objects mass, v is its velocity, and R is the radius of the circle.

5. If the radius of the circle is tripled, by what factor can the velocity be increased while maintaining the same force?

6. If a certain object can withstand four times the net force that it is currently experiencing, by what factor can its velocity be increased?

The force of gravity, F , between two objects with masses m_1 and m_2 is given by $F = K \frac{(m_1 m_2)}{r^2}$, where K is a constant and r is the distance between them.

7. If the distance between two objects decreases by a third, what is the ratio of the previous force of gravity between them to the new force of gravity between them?

8. The planet and moon in System A are each twice as large as the planet and moon in System B. If the forces of gravity in both systems are the same, what is the ratio of the distance between the objects in System A to the distance between the objects in System B?

Sum of Solutions

1. -7
2. $\frac{4}{3}$
3. 2
4. $\frac{1}{2}$
5. 4
6. $\frac{3}{4}$
7. -5
8. $\frac{8}{3}$
9. $\frac{10}{3}$
10. 8

Systems of Equations

1. -8.5
2. 6.5
3. $-\frac{4}{3}$
4. 6
5. $4\sqrt{3}$
6. $c = -15, k = \frac{10}{3}$

Equivalent Expressions

1. 3
2. 9
3. 5
4. 16
5. 7 and 14
6. 20

Dividing Rational Expressions

1. $3 + \frac{7}{2x-1}$
2. $7 + \frac{10}{x-1}$
3. $5 + \frac{36}{x-7}$
4. $4 - \frac{14}{2x-3}$
5. $1 + \frac{5}{3x-1}$
6. $4 - \frac{12}{3x+4}$

Factoring by Grouping

1. $(x^2 - 2)(x + 7)$
2. $(2x^2 - 3)(x - 3)$

3. $-4, -1, 1$
(factorization is $(x + 4)(x + 1)(x - 1)$)
4. $-5, \frac{1}{3}, 5$
(factorization is $(x + 5)(3x - 1)(x - 5)$)

Evaluating the Discriminant

1. 2 real solutions, crosses x -axis twice.
2. 2 complex solutions, never crosses x -axis.
3. 1 real solution, tangent to x -axis.
4. 6.25
5. 3
6. 6

Maximum/Minimum

1. 15
2. 6
3. 17
4. 10
5. 13
6. 29
7. 6
8. 41
9. 12

Sin/Cos and Tan/Cot

1. $.7$
2. $.35$
3. 68
4. 6
5. 90
6. 90
7. 1.5
8. $.93$
9. 60
10. 15

11. 90
12. 0

Percent Change

1. 49%
2. 600%
3. 53%
4. 21%
5. 6.9%
6. 17.5%
7. 38.6 inches
8. 705
9. 8.7%
10. 81%

Changing Parts of a Formula

1. 4 times
2. $\sqrt{2}$ times
3. P is tripled
4. $\frac{1}{6}$
5. $\sqrt{3}$
6. 2
7. $\frac{4}{9}$
8. $\frac{2}{1}$