Smart Math Strategy: Making Up Numbers

Some math questions are difficult because they are written in terms of variables rather than using all numbers. This is referring not to questions that ask for the value of a variable, **but rather where the question is written in terms of variables—**e.g., "If it takes *x* hours to make *y* parts in a factory…." The presence of variables instead of numbers can make algebraic steps more confusing, and also makes it tougher to use process of elimination to determine whether an answer seems reasonable.

If the challenge is the lack of numbers, one work-around is just to make up numbers and put them in place of the variables. It turns an algebra question into an arithmetic question and will likely be much easier to solve.

Use Making Up Numbers when the question is difficult because it's written in terms of variables, where those variables could have many different possible values.

<u>*Steps for Making Up Numbers :*</u>

1. Choose a different number for each variable in the question.
   - Use low positive numbers or numbers that fit the question, like 100 for a percent.
   - Do not choose 0 or 1, or any numbers that could make things complicated.
2. Replace *every* variable with numbers, everywhere you see them: in both the question **and** all of the answer choices.
3. Solve the resulting arithmetic question. (And/or, use process of elimination to get rid of unreasonable answers.)
4. If two or more answer choices yield the correct value, change your numbers.

Making Up Numbers Explanations

1. If $x - 1$ is a multiple of 3, which of the following must be the *next* multiple of 3?

(A) $x - 4$
(B) $x + 2$
(C) $2x - 2$
(D) $3x - 3$
(E) $3x - 1$

Making Up Numbers:
Making Up Numbers will work because the question and answers are both in terms of variables. There is only one variable, x, so we will choose a number to substitute for x. The only condition is that $x - 1$ is a multiple of 3, so we can choose numbers like $1, 4, 7, 10, 13$, etc. Let's choose $x = 4$ because something worth mentioning will happen later if we choose that number.

The next step is to convert all of the answer choices into numbers using $x = 4$:

(A) $x - 4$	$= 4 - 4$	= **0**
(B) $x + 2$	$= 4 + 2$	= **6**
(C) $2x - 2$	$= 2(4) - 2$	= **6**
(D) $3x - 3$	$= 3(4) - 3$	= **9**
(E) $3x - 1$	$= 3(4) - 1$	= **11**

Now that we've substituted $x = 4$ into the answers, we'll also substitute it into the question. The question now reads:

"If $4 - 1$ is a multiple of 3 (which is true, so we made up an appropriate number), which of the following must be the *next* multiple of 3?"

(A) 0
(B) 6
(C) 6
(D) 9
(E) 11

6 is the next multiple of 3. Here is that "something worth mentioning" we alluded to earlier: two of the answer choices, (B) and (C), *both* give us the number we're looking for. Rarely but occasionally this will happen. In this case, it's just a bit of bad luck that trying 4 happens to make (B) and (C) come out the same, *and* that one of those happens to be the right answer.

When this happens, eliminate the wrong answers and try another number with just the remaining possibilities. For example, we could now try $x = 7$:

(B) $x + 2$ $= 7 + 2$ $= 9$
(C) $2x - 2$ $= 2(7) - 2$ $= 12$

"If $7 - 1$ is a multiple of 3 (which is true, so we made up an appropriate number), which of the following must be the *next* multiple of 3?"

$7 - 1 = 6$, so the next multiple of 3 is 9, so the answer is (B).

Algebra:
We know that $x - 1$ is a multiple of 3 and we're looking for the next multiple of 3. If someone told you that 546 is a multiple of 3, how would you find the *next* multiple of 3? You'd simply add 3 to get 549. The same logic applies here: to find the next multiple of three in any situation, add 3. Take $x - 1$, add 3, and we get (B), $x + 2$.

Answer:
(B) $x + 2$

2. If $a = 3b$ and $b = 4c - 1$, then what is $a + 3$ in terms of c?

(A) $12c - 1$
(B)$12bc - 3b$
(C) $3b + 4c + 2$
(D) $12c - 3$
(E) $12c$

Making Up Numbers:
We must be careful to ensure that a, b, and c meet all of the conditions of the question. If we start with $a = 2$, then $b = \frac{2}{3}$ ...annoying. It's easier to start with c. If $c = 2$, then $b = 4(2) - 1 = 7$, and $a = 3(7) = 21$. That means $a + 3 = 24$.

We now plug our numbers into the answer choices and get (A) = 23, (B) = 147, (C) = 31, (D) = 21, (E) = 24. The answer is (E) since it matches 24.

Algebra:
If $b = 4c - 1$, substitute that into the first expression to get $a = 3(4c - 1)$. This simplifies to $a = 12c - 3$. We need $a + 3$, so add 3 to both sides to get $a + 3 = 12c$.

Answer:
(E) $12c$

3. A machine makes r objects in s minutes. Which of the following represents the number of objects made in 3 hours?

(A) $3rs$
(B) $\frac{3s}{r}$
(C) $\frac{180r}{s}$
(D) $\frac{180s}{r}$
(E) $\frac{20r}{s}$

Making Up Numbers:
We must choose values for each of the two variables. There are no particular units associated with r, so we choose a simple positive number like $r = 2$. On the other hand, s is in minutes and will be converted to hours, so it will make our math easier to choose $s = 60$. You don't *have* to choose 60, but it makes your arithmetic one step shorter if you do.

Now we convert the answer choices to numbers:

(A) $3rs \quad = 3(2)(60) \quad = 360$

(B) $\frac{3s}{r} \quad = \frac{3(60)}{2} \quad = 90$

(C) $\frac{180r}{s} \quad = \frac{180(2)}{60} \quad = 6$

(D) $\frac{180s}{r} \quad = \frac{180(60)}{2} \quad = 540$

(E) $\frac{20r}{s} \quad = \frac{20(2)}{60} \quad = \frac{2}{3}$

Now we plug our s and r values into the original question:

A machine makes 2 objects in 60 minutes. Which of the following represents the number of objects made in 3 hours?

(A) 360
(B) 90
(C) 6
(D) 54
(E) $\frac{2}{3}$

If the machine made 2 objects in 60 minutes, that's 2 objects in one hour, so it would make $3 \times 2 = 6$ objects in three hours. The answer is (C).

Algebra:
There is a comparison of units, so this is almost certainly a ratio and proportion question. We set up our units first, then the numbers and variables:

$$\frac{Objects}{Minutes} \qquad \frac{r}{s} = \frac{x}{180}$$

Because your unit is *minutes* you cannot put in 3 hours; you must convert it to minutes by multiplying by 60.

We cross-multiply and our math proceeds as follows:

$$\frac{r}{s} = \frac{x}{180}$$

Cross-multiply:

$$xs = 180r$$

Divide by s:

$$\frac{xs}{s} = \frac{180r}{s}$$

Simplify:

$$x = \frac{180r}{s}$$

Answer:
(C) $\frac{180r}{s}$

4. A certain bus stops every m minutes on average. If a bus travels for h hours, on average how many stops would it make?

(A) $\frac{h}{m}$
(B) $\frac{60h}{m}$
(C) $\frac{h}{60m}$
(D) $\frac{60m}{h}$
(E) $\frac{m}{60h}$

Making Up Numbers:
Since m is in minutes, we can make $m = 60$ for easy math. We'll make $h = 2$. You can use other numbers, but the calculations in this explanation will use $m = 60$ and $h = 2$. We convert the answer choices into numbers:

(A) $\frac{1}{30}$
(B) 2
(C) $\frac{1}{1800}$
(D) 1800
(E) $\frac{1}{2}$

Now we plug our numbers into the question: "A certain bus stops every 60 minutes on average. If a bus travels for 2 hours, on average how many stops would it make?" Obviously if the bus stops once per hour, it would stop 2 times in two hours, so the answer is (B).

Algebra:
Since this is a comparison of stops to time, we set up a proportion. The units are $\frac{stops}{minutes}$. Our proportion will be: $\frac{1}{m} = \frac{x}{60h}$. (In order to have the same units on both sides of the proportion, we must convert h hours into minutes by multiplying by 60.) We cross-multiply and solve for x:

$$\frac{1}{m} = \frac{x}{60h}$$

$$mx = 60h$$

$$x = \frac{60h}{m}$$

Answer:

(B) $\frac{60h}{m}$

5. Which of the following expressions represents the maximum number of regions into which a rectangle can be divided by x horizontal lines and y vertical lines?

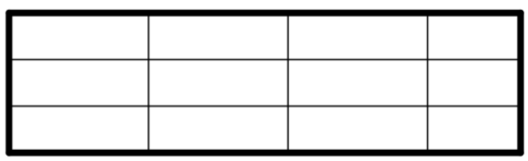
(A) xy
(B) $x + y + 2$
(C) $xy + 2$
(D) $2xy$
(E) $(x + 1)(y + 1)$

Making Up Numbers:
We can use easy numbers like $x = 2$ and $y = 3$. We then plug these numbers into the answer choices:

(A) 6 (B) 7 (C) 8 (D) 12 (E) 12

Now we plug our numbers into the question: "Which of the following expressions represents the maximum number of regions into which a rectangle can be divided by 2 horizontal lines and 3 vertical lines?"

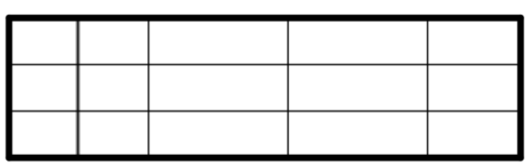
It's a geometry question, so immediately we'll try to draw it:



We can create 12 regions. Unfortunately, this matches two of the answer choices, (D) and (D), so we're not done. But we can easily change one of the numbers and try again. Let's change y from 3 to 4. First we recalculate (D) and (E):

(C) 16
(D) 15

And then we add one more vertical line to our drawing:



We can now count 15 regions, so we have our answer.

Answer:
(E) $(x + 1)(y + 1)$

6. A phone call is charged at c cents, which covers up to 3 minutes, and then x cents per minutes after the first three. What is the total cost in cents of an m-minute phone call?

(A) $c + mx$
(B) $mx - c$
(C) $c + x(m - 3)$
(D) $c + m(x - 3)$
(E) $cx + mx$

Making Up Numbers:
The answer choices are full of variables, so we have the option to make up numbers. The numbers don't have to be realistic, just easy. We can choose $c = 2$, $x = 3$, and $m = 4$.

The answer choices convert to:
(A) 14
(B) 10
(C) 5
(D) 2
(E) 18

Now we plug our numbers into the question:

"A phone call is charged at 2 cents, which covers up to 3 minutes, and then 3 cents per minute after the first three. What is the total cost in cents of a 4-minute phone call?"

The first three minutes cost 2 cents total, and the fourth minute costs 3 cents more. The total is 5 cents, so the answer is (C).

Algebra:
We can translate the question into a mathematical expression. The c cents for the first three minutes is easy enough. Then we could say that we're charged x cents per minute for *every minute except the first three*; in other words $x(m - 3)$. Add these together and we get the total cost, $c + x(m - 3)$

Answer:
(C) $c + x(m - 3)$

7. If $a < b < -1$, which of the following must also be true?

(A) $a + b > a - b$
(B) $a^2 > b^2$
(C) $2a > 2b$
(D) $-\frac{a}{2} < -\frac{b}{2}$
(E) $a^3 > b^3$

Making Up Numbers:
We must pick numbers for a and b that are true. The simplest numbers to choose would be $b = -2$ and $a = -3$. If we choose these or other appropriate numbers and plug them into the answer choices, we will find that only one of the answer choices yields a true statement — answer choice (B). The rest of the answers will yield nonsense statements like $3 > 4$.

Algebra:
We can also analyze each of these answer choices without using numbers. We must simply remember that a and b are both negative, with a being *more* negative.

(A) basically asks whether adding a negative number would yield a greater result than subtracting a negative number. The answer is no; subtracting a negative actually makes a number bigger, while adding a negative makes it smaller.

For (B), we first realize that a negative number squared becomes positive, so we're not worried about the signs in (B). We would then realize that squaring a "large" negative number would actually give us a very large positive result, while squaring a "small" negative number would give us a smaller positive result. (B) makes sense.

If we keep checking, (C) is not reasonable. We could divide both sides by 2 and get the result $a > b$, which we already know to be false.

We can simplify (D) by first multiplying both sides by two to get $-a < -b$. The best next step would be to multiply both sides by -1, being careful to switch the direction of the inequality sign since we're multiplying by a negative. The result would be $a > b$, which again we know to be false.

(E) is incorrect because when negative numbers are cubed, they'd stay negative. a^3 would be a "big" negative number, which would be less than, not greater than, a "small" negative number like b^3.

Answer:
(B) $a^2 > b^2$

8. A square has sides of length x, and a rectangle has width y and a length twice that of the square. If the rectangle has a larger area, what is the difference in area between the rectangle and the square?

(A) $y^2 - x^2$
(B) $2y - x$
(C) $xy - x^2$
(D) $x(2y - x)$
(E) $y(2x - y)$

Making Up Numbers:
We will choose a number for x and a number for y. For this example we'll use $x = 2$ and $y = 4$. We convert the answer choices to numbers:

(A) 12
(B) 6
(C) 4
(D) 12
(E) 0

The question now says: "A square has sides of length 2, and a rectangle has width 4 and a length twice that of the square...what is the difference in area between the rectangle and the square?"

The area of the square is $2 \times 2 = 4$. The area of the rectangle is $Area = length \times width$. The width is 4 and the length is twice the square's width — i.e. 2×2, so that's also 4. The rectangle's area is $4 \times 4 = 16$, the square's area is 4, so their difference is 12.

Given our numbers, both (A) and (D) are possibilities. We would have to recalculate, perhaps by changing to $y = 5$. Now the square's area is 4, and the rectangles area is $5 \times 4 = 20$. The difference is 16. Answer choice (A) $y^2 - x^2$ is now $5^2 - 2^2 = 21$, which is incorrect. Answer choice (D) $x(2y - x)$ is now $2(2 \times 5 - 2) = 16$, which is correct. The answer is **(D)**.

Algebra:
You can solve this question algebraically. If the square has sides of length x, then its area must be x^2. The rectangle has width y and length twice the squares length - in other words, length $2x$. The rectangle's area is $length \times width$, or $(2x)(y) = 2xy$. The difference in areas means to subtract the area of the square from the area of the rectangle: $2xy - x^2$. Finally, we can match the format of the answers by factoring out the greatest common factor x to get $x(2y - x)$. This matches answer choice (D).

Answer:
(D) $x(2y - x)$

9. A is the average of a group of n items, and one item in the group has a value of k. What is the new average when the item with value k is removed?

(A) $A - \frac{k}{n}$
(B) $\frac{A-k}{n-1}$
(C) $\frac{An-k}{n}$
(D) $\frac{A(n-k)}{n}$
(E) $\frac{An-k}{n-1}$

Making Up Numbers:
Almost any numbers will work but let's try to keep it simple: $A = 2, n = 3, k = 4$. We plug these numbers into the answer choices:

(A) $2 - \frac{4}{3} = \frac{2}{3}$
(B) $\frac{2-4}{3-1} = -1$
(C) $\frac{2\times3-4}{3} = 2/3$
(D) $\frac{2(3-4)}{3} = -2/3$
(E) $\frac{2\times3-4}{3-1} = 1$

We cannot eliminate answers because they're negative, fractions, duplicates, or whatever else. We have no clue what the actual answer is at this point. We simply plug our numbers into the question:"2 is the average of a group of 3 items, and one item in the group has a value of 4. What is the new average when the item with value 4 is removed?"

You need some knowledge of averages to solve this question. Using the average formula $Average = \frac{sum\ of\ items}{number\ of\ items}$, we can construct that $2 = \frac{sum\ of\ items}{3}$. Therefore the sum of the 3 items must be 6.

When the item with the value of 4 is removed, the remaining value is $6 - 4 = 2$. Now we can figure out the new average: the sum is 2, the number of items is 2 (now that one has been removed), so $Average = \frac{2}{2} = 1$. This matches answer choice (E).

Algebra:
We're removing one item from the group, so to find the new average, we must divide by $n - 1$, not by the original number of items n. This leaves only (E).

Answer:
(E) $\frac{An-k}{n-1}$

10. A group of a items has an average of p, and a group of $2a$ items has an average of r. What is the overall average when the two groups are combined?

(A) $\frac{(p+r)}{2}$
(B) $\frac{(p+2r)}{3}$
(C) $\frac{(2p+r)}{3}$
(D) $\frac{p+r}{3}$
(E) $\frac{(ap+2ar)}{3}$

Making Up Numbers:
This is a perfect question to use Making Up Numbers because the answers are complicated arrangements of variables. We will choose easy numbers, like $a = 3$, $p = 4$, and $r = 5$. We will plug these numbers into our answer choices:

(A) $4\frac{1}{2}$
(B) $\frac{14}{3} = 4\frac{2}{3}$
(C) $\frac{13}{3} = 4\frac{1}{3}$
(D) 3
(E) 14

The question now reads: A group of 3 items has an average of 4, and a group of 6 items has an average of 5. What is the overall average when the two groups are combined?

We can use process of elimination: if the groups have averages of 4 and 5, then their combined average must be between 4 and 5. We can eliminate (D) and (E). There are more items with an average of 5, so the average would be closer to 5 than to 4. The answer must be (B).

We can also figure out the exact average mathematically: $A = \frac{(3)(4)+(6)(5)}{3+6} = 4\frac{2}{3}$.

Algebra:
Because there are twice as many r's as p's, we must include twice the number of r's as p's in our average. Our average would be $A = \frac{(2r + p)}{3}$; we divide by 3 because we counted 2 r's and 1 p, or 3 total items. We can also construct a full average, $A = \frac{ap + 2ar}{a+2a}$, then simplify to get the answer. If you've already learned about complex averages, you should know how to do this, otherwise, it's not a big deal.

Answer:
(B) $\frac{(p+2r)}{3}$