Smart Math Strategy: Guess-and-Check

Some math questions allow you to simply check each answer to see which one works. We will call this strategy Guess-and-Check. Guess-and-Check will not work on every question, even when the answers are all numbers. ***You can use Guess-and-Check when the question gives you the output or finishing value, and asks you for the input or starting value.*** *In this situation, you can input the various answers and see which ones leads you to the correct final value (output).*

Steps

1. Identify the outcome you're looking for — for example, $8 in #1 on the next page.
2. On the SAT, start with answer (B) or (C) — whichever you think is more likely.
3. Input that value into the question and see if you get the outcome you identified in Step 1.
4. If yes, you're done. If no, but you can determine the right answer with certainty, you're done.
5. Otherwise: *write down the result you got next to the answer choice it came from.*
6. If necessary, try the next useful answer choice. The pattern of the results will tell you the answer in nearly every case.
7. Worst case scenario: just keep trying answers until one of them works!

Guess-and-Check Explanations

1. Joe has x dollars. He spends half, gets another \$10, spends half of what he has now, and ends up with \$8. Find x.

(A) 8
(B) 10
(C) 12
(D) 18

Guess-and-Check:
Start with either (B) or (C). If we start with (B), the calculation is: Joe starts with \$10; he spends half, so he's down to \$5; gets another \$10, putting him at \$15; and spends half of that amount, ending up at \$7.50. Since he needs to end up at \$8, this is too low. We can cross out (B), as well as (A), which is even lower. Going through the same process with answer choice **(C)** leaves Joe with \$8, the correct answer.

Algebra:
The easiest method is to work backwards. If Joe spends half of an amount to end up with \$8, he must have had \$16 before. He had \$16 after getting \$10 additional, so he had \$6 before that. He therefore had \$6 after the first time he spent half of his money, so he must have started with \$12.

An equation could also work: $\frac{\frac{x}{2}+10}{2} = 8$.

Solving this equation will yield $x = 12$

2. If the product of twice x and three less than x is 56, which of the following could be x?

(A) 7
(B) 8
(C) 9
(D) 10

Guess-and-Check:
This is a textbook example of when you can use the Guess-and-Check strategy: when a question asks for the value of x, gives you *numbers* as answer choices, and looks like it might take a while to solve using equations and algebra.

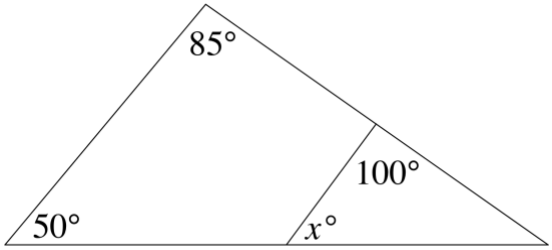
Every time we use the Guess-and-Check strategy, we start with either (B) or (C). If we plug in answer choice (B) first, the question reads: "If the product of twice 8 and three less than 8 is 56…."

Product means multiply, so we'll multiply "twice 8" and "three less than 8" - in other words, we'll multiply 16 and 5. That gets us 80; we're looking for 56, so clearly answer choice (B) is too high. [If we had started with choice (D), that would have given us 140, much too high, and we would have then tried answer choice (B).]

Since (B) is too high and the answer must be the only lower answer, **(A)**. We don't even need to check it.

Algebra:
Translate the question into an equation: $2x(x - 3) = 56$; then distribute to get $2x^2 - 6x = 56$. Since this is a quadratic (x^2) equation, we must make one side equal zero and then solve:

$$2x^2 - 6x - 56 = 0$$
$$2(x^2 - 3x - 28) = 0$$
$$(x^2 - 3x - 28) = 0$$
$$(x - 7)(x + 4) = 0$$
$$x = 7 \; and \; x = -4$$



3. Given the diagram above, what is x?

(A) 15
(B) 20
(C) 25
(D) 35

Guess-and-Check:
We start with either (B) or (C); let's start with (B). If x is 20, then $\angle C$ is 60°, since the small triangle must add up to 180°. But if $\angle C$ is 60°, then the larger triangle adds up to 195°, which is impossible.

So we know that 20 is wrong, but we don't know whether our real answer must be higher or lower. That's OK; let's try (C) and see what happens. Using the same series of calculations, if x is 25 then the larger triangle would add up to 190°. Our answer choices look like this:

(A) 15
(B) 20 195
(C) 25 190
(D) 35

A value of 180° can only be at the very bottom of this list, so the answer must be **(D)**.

Algebra:
The larger triangle must add up to 180°, and it already has angles of 50° and 85°. Therefore the third angle (in the lower-right corner) is $180° - 50° - 85° = 45°$. The small triangle also adds up to 180°, and it has angles of 100°, 45°, and x°. We set up $100° + 45° + x° = 180$ and solve for $x = 35$.

If your question #4 is about golf, see the next page.

4. Fred has \$1's, \$5's, and \$20's. He has fifteen bills and a total of \$119. If he has equal numbers of \$1's and \$20's, how many \$5's does he have?

(A) 7
(B) 9
(C) 11
(D) 13

Guess-and-Check
As always, we start Guess-and-Check with either (B) or (C). If we start with (B), we assume that Fred has 9 five-dollar bills. Since he has 15 total bills, he has $15 - 9 = 6$ other bills besides the fives. The question says that these are split evenly between ones and twenties, so there must be $6 \div 2 = 3$ of each. How much money would Fred have in this scenario? He would have $9(\$5) + 3(\$1) + 3(\$20) = \$108$. This is not the right answer, so we write down this number next to answer choice (B).

We don't know immediately whether we need more or fewer five-dollar bills, so we now try (C). We assume that Fred has 11 five-dollar bills. Since he has 15 total bills, he has $15 - 11 = 4$ other bills besides the fives. The question says that these are split evenly between ones and twenties, so there must be $4 \div 2 = 2$ of each. How much money would Fred have now? He would have $11(\$5) + 2(\$1) + 2(\$20) = \$97$. We write this next to answer choice (C). Here's how our answer choices look now:

(A) 7
(B) 9 \$108
(C) 11 \$97
(D) 13

If we're looking for \$119, it can only be at the the top of this list, so the answer is **(A)**.

Algebra:
More annoying to set up than is worth it. Where x is the number of five-dollar bills, the correct equation would be:

$$5x + 1\left(\frac{15 - x}{2}\right) + 20\left(\frac{15 - x}{2}\right) = 119$$

If your question #4 is about dollar bills, see the previous page.

4. In a certain 18-hole round of golf totaling 75 shots, a golfer only makes scores of 3, 4, and 5 on the various holes. If the golfer had equal numbers of 4's and 5's, on how many holes did the golfer score a 3?

(A) 4
(B) 6
(C) 8
(D) 10

Guess-and-Check:
We start with (B) by assuming that the golfer scored a 3 on six of the holes. This leaves 12 other holes. We are told that the golfer makes equal numbers of 4's and 5's, so she must have made six 4's and six 5's. Does this equal 75? No — six 3's, six 4's, and six 5's adds up to 72, so (B) is incorrect.

If we want to be clever, we can see that the golfer's score is too low, so she must need more high scores (4's and 5's) and fewer low scores (3's). The only answer choice where the golfer scores fewer 3's is answer choice (A).

If we want to be safe and thorough, we can try answer choice (C) and assume that the golfer makes eight 3's. Therefore the other eight holes are split with equal numbers of 4's and 5's — five of each. Eight 3's, five 4's, and five 5's adds up to 69.

Whether we started with (B) or (D), our answer choices would look like this:

(A) 4
(B) 6 72
(C) 8 69
(D) 10

Following the pattern, a value of 75 would have to be at the top of this list, so the answer must be **(A)**.

Algebra:
More annoying to set up than is worth it. Where x is the number of 3's the golfer scores, the correct equation would be:

$$3x + 4\left(\frac{18-x}{2}\right) + 5\left(\frac{18-x}{2}\right) = 75$$

5. What is x if the average (arithmetic mean) of 3, 17, and x is 19?

(A) 34
(B) 35
(C) 36
(D) 37

Guess-and-Check:
We start by assuming $x =$ either 35 or 36 (B or C). If we start with (B) 35, the average of 3, 17, and 35 is $18\frac{1}{3}$. This is too low, so we cross out (B) and (A). If we start with or then attempt (D) 36, the average of 3, 17, and 37 is $18\frac{2}{3}$. This is still too low, but getting closer, so the answer is **(D)** 37.

Algebra:
Use the average formula, $A = \frac{sum}{number\ of\ items}$:

$$19 = \frac{3 + 17 + x}{3}$$

Use algebra to solve for $x = 37$.

6. If $f(x) = x^2 + 3x - 8$, and $f(r) = 62$, what is r?

(A) 4
(B) 5
(C) 6
(D) 7

Guess-and-Check:
Start with either (B) or (C); let's try (C) this time just for variety. $6^2 + 3(6) - 8$ is 46. It's too low, and there's only one higher value, so that is that only possible answer. That's all we have to do; the answer is **(D)**.

Algebra:
This question will be very difficult to solve if you don't have a basic idea of what a function is, no matter what technique you use. Hopefully you understand that in this case, the question is saying that $r^2 + 3r - 8 = 62$. Since it's a quadratic (x^2) equation, we must make one side equal zero, factor, and solve as follows:

$$r^2 + 3r - 8 = 62$$
$$r^2 + 3r - 70 = 0$$
$$(r - 7)(r + 10) = 0$$

$$r - 7 = 0 \quad and \quad r + 10 = 0$$
$$r = 7 \quad and \quad r = -10$$

Only 7 is one of the answer choices, so the answer is **(D)**.

7. Given that $a = \frac{b}{3} + 1$, $b = \frac{c}{2} + 2$, and $a = 5$, what is c?

(A) 16
(B) 18
(C) 20
(D) 22

Guess-and-Check:
If we start with (B) by assuming that $c = 18$, then $b = \frac{c}{2} + 2$. Therefore $b = \frac{18}{2} + 2$, so $b = 11$. We're also told that $a = \frac{b}{3} + 1$, so $a = \frac{11}{3} + 1$, and we solve for $a = 4\frac{2}{3}$. But the question says $a = 5$; this means that answer choice (B) is not correct. Since our result was too low, we'll cross off (B) and (A) and now try (C).

Using (C), $c = 20$, then $b = \frac{20}{2} + 2$, so $b = 12$. We're also told that $a = \frac{b}{3} + 1$, so $a = \frac{12}{3} + 1$, and we solve for $a = 5$, as we hoped. The answer is **(C)**.

Algebra:
$a = 5$ and $a = \frac{b}{3} + 1$, so $5 = \frac{b}{3} + 1$. Solve this for $b = 12$. Also, $b = \frac{c}{2} + 2$, so $12 = \frac{c}{2} + 2$. Solve this for $c = 20$.

8. Charlie purchased 7 video games for a total cost of $270. If all of the games cost either $35 or $40, how many $40 video games did Charlie purchase?

(A) 4
(B) 5
(C) 6
(D) 7

Guess-and-Check:
We can start with (B) by assuming that Charlie purchased five $40 video games. Since he purchased seven games total, he would also have purchased two $35 games. Five $40 games and two $35 games would add up to $270, so **(B)** is the correct answer.

If we had started with with answer choice (C), six $40 games and one $35 game would yield a total cost of $275. This is too high, so the correct answer would include fewer of the more expensive games. We would then try answer choice (B).

Algebra:
Let $x = \textit{the number of \$40 games}$ and $y = \textit{the number of \$35 games}$. We have one equation for the number of games, $x + y = 7$. We have a second equation for the total cost, $40x + 35y = 270$.

Use substitution: solve for y in terms of x to get $y = 7 - x$. Substitute this value into the second equation to get $40x + 35(7 - x) = 270$, and solve for $x = 5$.

9. Sarah is 3 inches taller than Camille. Marlyn is 6 inches shorter than Louise and 5 inches shorter than Camille. If Louise is 68 inches tall, how tall is Sarah?

(A) 69 inches
(B) 70 inches
(C) 71 inches
(D) 72 inches

Guess-and-Check:
We start with (B) by assuming that Sarah is 70 inches tall. Sarah is 3 inches taller than Camille, so Camille must be 67 inches. We do not have enough information to use the information about Marlyn and Louise yet, but we are told that Marlyn is 5 inches shorter than Camille. Camille is 67 inches, so Marlyn is 62 inches. Now we can use the information that Marlyn is 6 inches shorter than Louise. Marlyn is 62 inches, so Louise is 68 inches. The question says that Louise actually *is* 68 inches tall, so answer choice **(B)** is correct.

If we started with (C), we would have found that Louise is 69 inches tall. This would be too high a number, so we would have eliminated (D) and tried answer choice (B) next.

Algebra:
A system of four simple equations — easy to solve with substitution, but not a very efficient way to solve this problem.

10. The average age of a group of four people is 49 years. If one of the people is 40 years old, what is the average age of the other three people?

(A) 52
(B) 54
(C) 56
(D) 58

Guess-and-Check:
We start by assuming that the average age of the other three people is (B) 54 years. That means the average age of all four people is

$$A = \frac{40 + 54 + 54 + 54}{4}$$

$$A = 50.5$$

But we know the actual average should be 49 years, not 50.5 years. Since (B) is too high, the answer can only be (A) 52.

If we had started with (C), we would have obtained an average of 52 — too high — and then tried (B) as described above.

Algebra:
The average age of one 40 year old person and 3 people with an unknown average age is 49. We can use the average formula, $A = \frac{sum}{number\ of\ items}$, and set up an equation representing a total of four people:

$$49 = \frac{40 + 3x}{4}$$

Solving this equation for x yields $x = 52$.

11. The height in inches, h, of one plant d days from now can be represented as $h(d) = 4 + .07d$. The height of a second plant in inches, k, can be represented as $k(d) = \sqrt{d} + 1.12$. After how many days will the two plants be the same height?

(A) 16
(B) 18
(C) 20
(D) 22

Guess-and-Check:
Start with (B). After 18 days, the first plant is $4 + .07(18) = 5.26$ inches tall. The second plant is $\sqrt{18} + 1.12 \approx 5.36$ inches tall. So we write:

(A) 16
(B) 18 5.26 5.36
(C) 20
(D) 22

It's incorrect, but we don't know where the correct answer will be, so we do the same thing with answer choice (C), obtaining the following results:

(A) 16
(B) 18 5.26 5.36
(C) 20 5.4 5.59
(D) 22

We want them to be the same height, but as we moved from (B) to (C), they actually got more different: in (B) they were 0.10 apart, but in (C) they're 0.19 apart. That means we need to go in the other direction, and the answer must be (A).

Algebra:
Set the two expressions equal, $4 + .07d = \sqrt{d} + 1.12$. Isolate the radical, square both sides, set one side equal to zero, and use the quadratic formula (with a calculator).

12. Given $f(x) = \frac{4x}{x^2-3}$, for which of the following values of a is $f(a) = f(3)$?

(A) -4
(B) -3
(C) -2
(D) -1

Guess-and-Check:
Solving this question can only start by finding the value of $f(3)$, so we set up $f(3) = \frac{4(3)}{3^2-3} = 2$. Which other number can we put into the function to get a result of 2? Starting with (B): $f(-3) = \frac{4(-3)}{(-3)^2-3} = -2$, so that doesn't work. But do we need to go higher or lower? We're not sure, so we try (C): $f(-2) = \frac{4(-2)}{(-2)^2-3} = -8$, so we seem to be getting farther from the answer. That means it's (A), right? Nope — (A) would give us $f(-4) = \frac{4(-4)}{(-4)^2-3} = -\frac{16}{13}$, which doesn't help at all. The correct answer is (D), $f(-1) = \frac{4(-1)}{(-1)^2-3} = 2$.

So what happened here? Aren't the answers supposed to give us results in order? Well…usually. In a simpler type of question, for example, if we start a scenario with more money, we probably end up with more money. But this question isn't so simple: there are quadratics involved, in a denominator, with a variable in the numerator too.

The lesson from this question: if you're doing Guess-and-Check and you have *any* uncertainty about what the answer is, just keep trying until you know for sure!

Algebra:
Begin by finding $f(3) = 2$. Then set up $\frac{4x}{x^2-3} = 2$ (or $\frac{4a}{a^2-3} = 2$ if we want to use the same letter as the question does). Multiply both sides by $x^2 - 3$. We see it's a quadratic so we get one side equal to zero and factor or use quadratic formula. We get $x = -1$ and $x = 3$, so the answer is (D) -1.

$$|2x - 5| > x + 3$$

13. Which of the following is NOT a solution to the equation above?

(A) 0
(B) 5
(C) 10
(D) 15

Guess-and-Check:
Start with (B): $|2(5) - 5| > 5 + 3$, so $5 > 8$. Since that statement is false, (B) is not a solution, and therefore is the right answer to this question.

If we started with (C): $|2(10) - 5| > 10 + 3$, so 15>13. That's a true statement, so it's not the answer.

The catch with this question is that, with absolute value, starting with a lower number might actually give us a higher end result. (The same thing applies with quadratics.) That makes it hard to tell whether we need to go up or down to find the right answer choice. Therefore we'd just keep trying choices until we find the right answer.

Any other answer choice besides (B) would give us a true statement, meaning it would be a solution and therefore not the right answer to this question.

Algebra:
Absolute value questions are solved by setting up two equations:

$$2x - 5 > x + 3$$
$$x > 8$$

$$-(2x - 5) > x + 3$$
$$2x - 5 < -x - 3$$
$$3x < 2$$
$$x < \frac{2}{3}$$

So the solutions occur where $x < \frac{2}{3}$ or $x > 8$. The only answer choice that is not in either of these ranges is (B).

$$\sqrt{x+18}+6=x+4$$

14. What is the solution set to the equation above?

(A) $\{-2\}$
(B) $\{7\}$
(C) $\{-2, 7\}$
(D) $\{2, 7\}$

Guess-and-Check
When you see a radical equation, you might as well use Guess-and-Check. Why? Because when you solve radical equations algebraically, you always have to check for extraneous solutions. As long as you'll have to check your answers anyway, why not just start that way?

In this case, starting with -2 is best because it appears in half of the choices: $\sqrt{-2+18}+6=-2+4$. That gives us $\sqrt{16}+6=2$, so $4+6=2$, which is certainly not true. Since -2 doesn't work, (A) and (C) are eliminated. The difference between (B) and (D) is whether 2 works, so let's try it: $\sqrt{2+18}+6=2+4$, so $\sqrt{20}+6=6$. That's false, so 2 doesn't work. Since neither -2 nor 2 works, the answer can only be (B).

As often happens with Guess-and-Check, we found the right answer without ever trying the actual number.

Algebra:

$$\sqrt{x+18}+6=x+4$$

Isolate the radical:

$$\sqrt{x+18}=x-2$$

Square both sides:

$$x+18=(x-2)^2$$

Expand:

$$x+18=x^2-4x+4$$

Set equal to 0:

$$0=x^2-5x-14$$

Factor:

$$(x-7)(x+2)=0$$

Set $=0$ and solve:

$$x=7, x=-2$$

Since it's a radical equation, we *still* need to check for extraneous answers, so we have to try 7 and -2 in the original radical equation anyway, just as if we were doing Guess-and-Check.

$$\frac{-4+a}{-4-a}=-\frac{1}{3}$$

15. Given the equation above, which of the following could be a?

(A) 2
(B) 4
(C) 6
(D) 8

Guess-and-Check:
Start with (B):

$$\frac{-4+4}{-4-4}=-\frac{1}{3}$$

$$\frac{0}{-8}=-\frac{1}{3}$$

False, so we try (C):

$$\frac{-4+6}{-4-6}=-\frac{1}{3}$$

$$\frac{2}{-10}=-\frac{1}{3}$$

Closer, so it's probably (D), but let's try it to make sure:

$$\frac{-4+8}{-4-8}=-\frac{1}{3}$$

$$\frac{4}{-12}=-\frac{1}{3}$$

True, so that's the right answer.

Algebra:

$$\frac{-4+a}{-4-a}=-\frac{1}{3}$$

We solve proportions by cross-multiplying:

$$3(-4+a)=-1(-4-a)$$

Distribute:

$$-12+3a=4+a$$

Solve:

$$2a=16$$
$$a=8$$

16. The perimeter of a rectangle is 24, and its area is greater than 30. If all of its sides are integer lengths, what is the greatest possible length of one of its sides?

(A) 7
(B) 8
(C) 9
(D) 10

Guess-and-Check:
We're looking for the greatest possible length, so it makes sense to start from the highest numbers and work our way down. Starting with (D):

It's a rectangle, so if one side is 10, the side across from it is 10. That adds up to 20. That means the other two sides add up so $24 - 20 = 4$, so each of those sides is 2. If the sides of the rectangle are 10 and 2, the rectangle's area is 20. It's supposed to be greater than 30, so that doesn't work.

Now we'll try (C): If one side is 9, following the same process, the other side is 3. That would give us an area of 27, still too small but getting closer.

Now (B): If one side is 8, the other side is 4. That gives us an area of 32, which is above 30, meaning this is the correct answer.

Algebra:
We use the formula for perimeter and set it equal to 24: $2L + 2W = 24$. Then we use the formula for area and set it to be greater than 30: $LW > 30$. We can solve this system of equations:

$$2L + 2W = 24$$
$$LW > 30$$

Get L in terms of W:

$$2L + 2W = 24$$
$$2L = 24 - 2W$$
$$L = 12 - W$$

Substitute:

$$(12 - W)(W) > 30$$
$$12W - W^2 > 30$$

Get one side equal to 0:

$$0 > W^2 - 12W + 30$$

It's not factorable, so we'd have to use the quadratic formula to find the zeroes (~3.55 and ~8.45), then interval test to find the range of solutions (it's between those numbers). The highest of our answer choices between those numbers is 8.