

2024 Test Prep Binder



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ACT Scoring System

<http://www.actstudent.org/scores/understand/>

Raw Score

- +1 for each right answer
- 0 for each wrong answer or blank

Raw points are converted to *scaled scores*. Each question in the English and Math sections is worth about $\frac{1}{2}$ a point to your scaled score for that section. Each question in the Reading and Science sections is worth about a point to your scaled score for that section.

Guessing

There is no penalty for wrong answers, so never leave a question blank. Fill in blanks at the end of each passage, and with 2-3 minutes to go on each section to ensure you do so without any errors. Then, go back to what you were doing.

The Tryout Section

Following the Science section, standard-time testers will receive a 20-minute Tryout Section. This section does NOT count towards your score. ACT uses this section to try out potential future question types and new material on real students. Unfortunately, there is no way around this section. My advice is to try to get these questions right, because zoning out for 20 minutes isn't going to help you be ready for the essay. But don't try too hard. Think of yourself as an athlete staying loose on the sidelines until you get back in the game.

Scaled Scores

- **Test Scores, out of 36:** Each of the five tests is scored on a 1-36 scale.
- **Composite Score, out of 36:** the rounded average of English, Math, Reading, and Science. It does not include the Writing score.
- **Writing Score, out of 12:** The average of the four "domain scores" listed below.
- **Writing Domain Scores, out of 12:** Two readers each grade four separate domains — *Ideas and Analysis, Development and Support, Organization, and Language Use* — on a 1-6 scale, so scores for each domain range from 2-12.
- **ELA, out of 36:** English-Language Arts, the average of English, Reading, and Writing.
- **STEM, out of 36:** the average of your Math and Science scores.

Colleges that require the Writing section may have varying systems for incorporating it into your score. See the college, university, or system's website for more information.

Information for the Digital SAT

Structure

[From College Board](#): The digital SAT is composed of two sections: Reading and Writing and Math. Students have 64 minutes to complete the Reading and Writing section (32 minutes and 27 questions per module) and 70 minutes to complete the Math section (35 minutes and 22 questions per module) for a total of 2 hours and 14 minutes.

Each section is divided into 2 equal length modules, and there is a 10-minute break between the R+W and Math sections. The first module of each section contains a broad mix of easy, medium, and hard questions. Based on how students perform on the first module, the second module of questions will either be more difficult or less difficult. Questions within each module are in rough order of difficulty.

Scoring

Generally, within each section, the first module establishes your approximate scoring level and the second module hones that into a precise number. Your score is not simply based on the number of questions you get right, because the better you do on the first module, the harder your questions will be in the second module.

Example 1

On the first R+W module, Student A gets 15 questions right and student B gets 25 questions right. On the second module, Student A gets 25 questions right and Student B gets 15 questions right. They both got 40 right total. Do they get the same score?

Answer: No. Student B performed better on the first module, so they will get tougher questions on the second module, and Student A will get easier questions. The two students may have gotten the same total number of questions right, but Student B's questions were harder overall, so they will receive a higher score.

Example 2

On the first Math module, Student C gets 19 questions right and Student D gets 21 questions right. Since they both did well, they both get the hard second module. On the second module, both students get 15 questions right. Do they get the same score?

Answer: No. Since the two students had the same difficulty level of questions, but Student D got 2 more right answers overall, Student D's score will be slightly better.

Guessing and Wrong Answers

There is no penalty for wrong answers. Never leave a question blank on the digital SAT.

How to Answer Multiple-Choice Questions

These are basic techniques that work on *all* multiple choice tests, not just on the ACT.

You must recognize that incorrect multiple choice answers are not just wrong; they are purposefully designed to be misleading. They are designed to appear like they *could* be right.

If you read a question and then immediately start browsing the answers, you are putting yourself right in the test-writer's trap: *all* of the answers are designed to *appear* correct! Understand that, depending on the number of answer choices, 75% or 80% of the answer choices on every question are both wrong and purposefully misleading. Why would you intentionally let them all swim around in your head?

It is much easier to find something when you know what it looks like and then go try to find it. Similarly, it's much easier to find a correct answer if you figure out what it looks like first and then go search for a match in the answer choices.

The single most important strategy for any multiple-choice test is:

Develop your own answer first. Then look at the answer choices.

This is **Strategy #1**. Why does it work? For several reasons:

- When you see the right answer, it will jump out at you.
- Wrong answers will clearly not match; you can eliminate them quickly.
- You eliminate confusion by keeping incorrect answers out of your thought process.
- You move more quickly — find what you're looking for, then move on.
- You'll get more questions right.

If this does not get you to the correct answer, the next step is process of elimination. Process of Elimination (POE) means eliminating answers that have something — anything! — clearly incorrect. Just like the True/False questions you've been doing since first grade, *if any part of an answer is incorrect, the entire answer is incorrect*. This is true even if just one word is wrong.

To summarize: Develop your own answer first. Then, if necessary, use POE.

Finally, it is worth noting that not every question can be approached this way. Questions like "Which of the following..." for example, might require you to look at the answer choices first.

Time Management

Imagine that you were given the following instructions:

“Ten twenty-dollar bills have been numbered one through ten and placed in a variety of locations. One or two are in this room; one or two are in nearby rooms; some are outside but nearby, and others are across town. You have ten minutes to collect as much money as you can. Ready...go!”

What would you do? Hopefully, you’d focus on the nearby “easy money” first and only worry about the more difficult-to-collect money once the easy stuff is already in your pockets. This is how you should approach questions on the ACT. They all have the same value — one point — but some questions will be much harder to get, and others will be easier. **On the ACT, SAT, and any other test of this type, focus on getting the easy questions right first.** Then, if you have time, do the medium questions. Finally, if you still have time, try the hard ones. It is always better to skip and come back than to spend too much time

The ACT is specifically constructed to be difficult to finish on time; the SAT is easier for some people, but know your own timing. Do not allow yourself to get stuck on a hard question if there are still easy ones that you haven’t done!

One final note: it doesn't matter whether a question is *supposed* to be easy or hard. If it's hard for you, it's hard, and you should consider skipping it and coming back later.

Extended Time

The ACT (*note: extended-time testers do not take the Experimental/Tryout section*):

50% extended time:

- English — 70 minutes
- Math — 90 minutes
- Break — 15 minutes
- Reading — 55 minutes
- Science — 55 minutes
- Break — 5 minutes
- Writing — 60 minutes

100% extended time:

- English — 90 minutes
- Math — 120 minutes
- Reading — 70 minutes
- Science — 70 minutes
- Writing — 80 minutes

The SAT:50% extended time:

- Reading — 98 minutes
- Break — 5 minutes
- Writing and Language — 53 minutes
- Math (No Calculator) — 38 minutes
- Break — 5 minutes
- Math (w/ Calc.) — 83 minutes
- Break — 1 minute

100% extended time:

- Reading — 130 minutes
- Writing and Language — 70 minutes
- Math (No Calculator) — 50 minutes
- Math (w/ Calc.) — 110 minutes

Resources

The Resources Page

- baytutoring.com/resources: Bookmark it now! Digital documents, links, answer explanations, practice tests, and more.

The ACT

- The Real ACT Prep Guide: 4 practice tests, full answer explanations, complete info)
- Your binder (includes comprehensive information and dozens of questions/answers)
- actstudent.org: includes 2 full practice tests, tips, \$20 option for more online prep
- ACT Academy, academy.act.org: a collection of videos and questions on ACT topics.

The SAT

- The Official SAT Study Guide: information about the test, content review, and 8 practice tests.
- Your binder (includes comprehensive information and practice questions/answers).
- Collegeboard.org: lots more SAT guidance, practice tests and questions.
- Ivy Global's online scoring system for the official practice tests (8 book/6 online). Use in real time or after a timed section or test. *Log in before you do anything!*
- Khanacademy.org/sat: the official practice source for the new SAT.

Other Test-Specific Resources

- Naviance (depending on whether your school purchased the test-prep component)
- Other books (such as Kaplan, Princeton Review, Barron's, etc.)
- Sparknotes.com (surprisingly comprehensive)

Content-Based Online Resources

- Kutasoftware.com (question-and-answer math worksheets on every topic)
- Aleks.com and AdaptedMind.com (math, modest monthly fees)
- Mathisfun.com and Purplemath.com (math instruction, free)
- Chompchomp.com (great site for grammar practice and quizzes)
- http://grammar.ccc.commnet.edu/grammar/ (more grammar and quizzes)
- Purdue OWL (for English/grammar and essay writing)
- IXL.com for math and grammar
- Google search for additional online resources, including YouTube videos.

How to Study

What is "Active Studying?"

Active studying is the basis of successful academic learning. When I say study or practice, I mean *do* questions and drills, from either this workbook, your SAT book, or another source. "Looking over" material is categorically ineffective.

I just did some questions. Now what?

Immediate feedback is the fundamental technique for initial learning. You obtain immediate feedback by diligently checking answers and reading answer explanations within minutes after completing a question, passage, or section. You must know if you do questions correctly so that your brain can either ingrain or quickly dismiss the process you used. **Check your answers diligently, immediately.**

The definition of "immediate" depends on the type of practice you do. Balance these types:

- Timed: do one section timed; review answers and explanations upon completion.
- Untimed: read answer explanations and do additional practice after every question or passage. This type of practice helps to build your skills; correct comes before fast!
- Paused: start a timer; do a few questions or a passage; pause your timer; check answers and explanations and do extra practice; restart your timer and continue. Do not exceed total allowed time for each section.
- Full Timed Test: do a full practice test under test conditions: timed, all in one sitting, early on a weekend morning, sitting at a desk or table, no distractions.

How do I get better at something?

Read answer explanations for every question. If you got it right, did you do so the best way? If you got it wrong, what didn't you know? **Practice missed concepts until you master them**; that's learning. If you do a question, miss it, and simply move on, you'll have learned nothing and wasted your time.

How can I remember what I learn?

Spaced repetition is the fundamental technique for placing items into long-term memory. To place information into long-term memory, you must reinforce your learning. Consistency and repetition create long-term memories. For this reason, **our expectation is that you will find time for ACT/SAT prep work 4+ days each week.**

SAT: Two Things About the Test

Context is meaningful.

College board attempts write questions that are context-based. It's an important part of their philosophy about what they want to test. For example, the Writing and Language questions refer to passages and emphasize relationships with the tone and content of the passage, while Reading questions ask you to understand how textual elements function relative to the overall meaning or purpose of the passage.

To answer context-dependent questions, you should always follow two basic principles:

1. Don't answer questions without considering the passage, chart, graph, etc. — whatever information is present. Many questions are *purposefully designed* so that you need this additional information!
2. Context-dependent questions can also make things easier for you: you can extrapolate the meaning of a word from how it's used in a sentence or passage; you can figure out appropriate phrasing from the tone of the rest of the passage; and so on.

There are lots of graphs and charts in all sections.

If you have already prepared for the ACT, you are mostly ready for this element of the SAT. Data representations won't be as complicated as those in the ACT Science section.

Either way, the process you must follow is the same: **always evaluate a graph or chart by first looking at its title, axes, units, scales, and keys.** Any representation in a graph or chart is meaningless without knowing what it actually shows. Once you understand the title, axes, units, scales, and keys, you can see what the graph or chart actually shows.

In other words: look at everything else first and the actual data last.

UNIT 1

ACT: Approaching the English Test

[English on actstudent.org](http://actstudent.org)

The five passages in the English test are not in any particular order. There is no order of difficulty or topics among the questions or passages, and there are no different types of passages. For standard-time students, learn to pace yourself to finish this test on time. Practice by doing individual passages in 9 minutes or less.

What's on the English Test?

- 40 questions about usage and mechanics — i.e., grammar and punctuation
- 35 questions testing your rhetorical skills — word choice, sentence construction, etc.

Basic grammar and punctuation rules are important.

You'll be asked many questions about fundamental grammar topics like verb tense, parallel construction, subject-verb agreement, run-on sentences, and pronouns, and about proper punctuation use with commas, apostrophes, dashes, colons, and semicolons.

Questions about organization, phrasing, and "sound" follow rules, too.

Don't think that you just have to go by "sound" on these questions. There are strategies for figuring out where a sentence should be placed within a paragraph; which phrase is the most relevant; and so on. There are clear, learnable techniques for all ACT question types!

How should you do ACT English passages?

1. Start reading the passage. Never skip any portion of the passage.*
2. When you get to a question, keep reading until the end of the sentence.
3. At the end of each sentence, answer any questions you passed.
4. Some questions, like "Should this sentence be deleted?", can't be answered until you've read the whole passage. If you see one of these, circle it and come back later.

As you read...

5. Notice any paragraphs with numbered sentences; they will be in brackets like [1], [2], and so on. This means there will *definitely* be a question about the ordering of sentences in that paragraph, or about where to insert a new sentence.
6. Err on the side of re-reading more than necessary — another few words of reading might give you the answer you need.

*If you know that you have time to read the whole passage first, do it! You can mark anything that sounds weird along the way. Then follow these steps. If you read the entire passage first, begin by answering the questions about the passage as a whole.

SAT Reading and Writing

Structure

Students have 64 minutes (standard time) to complete the Reading and Writing section (32 minutes and 27 questions per module). The section includes a mix of Reading (paragraphs, graphs, and charts with one question each) and Writing (grammar, punctuation, vocabulary usage, and other conventions of effective writing).

How to Approach Questions

Because there is only one question per passage, graph/chart, or sentence, you can be purposeful and precise in how you approach each one.

1. Read the question first. Since the answer that single question is the *only* thing you'll ever need to know about this passage, reading the question first allows you to be very clear about what is important to focus on (and what is not).
2. That said—consider the entire context in answering questions. The SAT has always preferred to ask context-based questions. Do not skip any portion of a passage, and do not think that the answer to a particular question relates only to the spot where that idea appears. Generally, when the SAT includes a lot of information, it's because a lot of information is relevant.
3. Look for known question types, SAT-favorite grammar and punctuation rules, and so on. Most questions don't need to be approached from scratch.

English/Writing and Language: Sentence Order and Transitions
[Building paragraphs \(2.X topics\)](#), [Arranging sentences](#), [Transitions](#)

Inserting, Moving, or Deleting Sentences Within a Passage

Many students find these questions tricky, but you can actually utilize many clues provided by the passage itself. A few of these are:

- Notice sentence or paragraph numbers in a passage. Sentence and paragraph numbers are only included when there will be a question about their order later on. As you're reading within a numbered section, notice if information comes out of nowhere or something is unexplained.
- The main strategy is to re-read the sentence in question and imagine for yourself what information you would expect to see both before and after. Then go find that information in the passage. Only as a last resort should you try inserting the sentence into each of the possible locations in the passage.

Transition Questions

Your brain automatically interprets what you read based on the transition words in the sentence. That's great — *except on SAT/ACT questions, where the given transition words might be incorrect*. To get these questions, train yourself to ignore those transition words:

1. Re-read the sentences before and after the underlined portion, skipping the underlined portion itself.
2. Decide for yourself how the sentences relate to each other — specifically, if they:
 - are cause-and-effect;
 - support each other; or
 - contradict each other.
3. Pick the answer choice that matches what you decided.

Example:

Participants in experiments have a right to know the purpose of the experiment, but telling them while they are participating might affect those participants' actions. **1** Additionally, standard procedure is to provide participants with this information only after the experiment's conclusion.

1. (A) NO CHANGE
 (B) However
 (C) In spite of this
 (D) Therefore

The first sentence says that telling them *during* the experiment would create problems. The second sentence says that they are provided with the information *after*. The first sentence is the cause, and the second sentence is the effect. This makes the answer (D) Therefore.

Transitions Practice

The sunk-cost fallacy refers to a common logical mistake, **1** although it's best illustrated with an example. Imagine that you prepaid \$50 for a nonrefundable outdoor concert ticket. On the day of the concert, **2** though, it's cold and wet. You'd rather not go — **3** by contrast, you're behind on your homework — **4** because you already spent the \$50 and you don't want to waste it. Should you go or stay home?

5 Whereupon this sounds like a subjective question, it's not: there is a right answer, which is that you should stay home. Why? Your \$50 is gone no matter what you decide: it's already "sunk", and you're not getting it back. Neither going nor staying at home costs you any additional money — **6** in other words, they're both free at this point — so you should do whichever activity would you truly prefer. **7** Alternatively, going to the concert wouldn't get your \$50 back, and you'd be less happy than if you'd stayed home.

8 Alternatively, the sunk-cost fallacy can be summarized as follows: *do not base future decisions on previously invested, unrecoverable resources*. This idea actually applies to standardized testing as well. **9** For example, imagine you've already spent 3 minutes on a math question. Should you move on or stick with it? **10** Despite our understanding of the sunk-cost fallacy, we know exactly how to think about this situation: you should ignore the time you've already "sunk" into it. **11** Instead, you should consider only whether, *looking only forwards*, you would get this question or some other question right more quickly.

1. (A) NO CHANGE
(B) and
(C) meaning that
2. (A) NO CHANGE
(B) furthermore
(C) as a result
3. (A) NO CHANGE
(B) nonetheless
(C) and besides
4. (A) NO CHANGE
(B) but
(C) given that
5. (A) NO CHANGE
(B) Just as
(C) Although
6. (A) NO CHANGE
(B) moreover
(C) conversely
7. (A) NO CHANGE
(B) Essentially
(C) Similarly
8. (A) NO CHANGE
(B) Notably,
(C) More generally
9. (A) NO CHANGE
(B) First of all
(C) Thus
10. (A) NO CHANGE
(B) Along with
(C) Based on
11. (A) NO CHANGE
(B) Finally
(C) Consequently

Punctuation: Semicolons, Colons, and Commas

[GB1](#), [GB2](#), [CCC 1](#), [CCC 2](#), [IXL 1](#), [IXL 2](#), [IXL 3](#), [Chomp Chomp 1](#), [Chomp Chomp 2](#)

Semicolons

Semicolons (;) separate complete sentences; if you see two related, complete sentences, you can separate them with a semicolon, just as you would with a period. (Semicolons can also be used to separate items in a list when each individual item is long or has commas in its description; however, this usage has not been tested recently):

- I like dogs, cats, and birds.
- I like cute, happy dogs; friendly, adorable cats; and squawking, feathered birds.

Colons

Colons (:) have two rules:

1. A colon can only come after a complete sentence.
 2. A colon acts like an = sign: what comes after it must "equal" what comes before it.
- I need three things to live: food, water, and air. (The three things = food, water, air.)
 - ~~Three things I need are: food, water, and air.~~ (Not complete sentence before colon.)
 - I left for one reason: I really had to pee. (The one reason = I had to pee.)

Commas

Commas have many rules, but one is that a comma *and a conjunction* (and, but, or, etc.) can separate two complete sentences. *A comma alone cannot separate complete sentences.*

Insert a semicolon, colon, or comma in each sentence below.

1. Bob said that he agreed with the idea__ David said that he was against it.
2. There was only one reason I agreed with Bob__ he had thought about it for years.
3. David didn't like this__ and he stated his dissatisfaction very clearly.
4. I didn't mind__ but I wish he had been more polite about it.
5. I thought about what to do and realized I had three choices__ confront him, have a friend talk to him, or stay silent.
6. Eventually I decided not to talk to him__ it wasn't worth the trouble.
7. Still, it left a bad taste in my mouth__ I never looked at him the same way again.

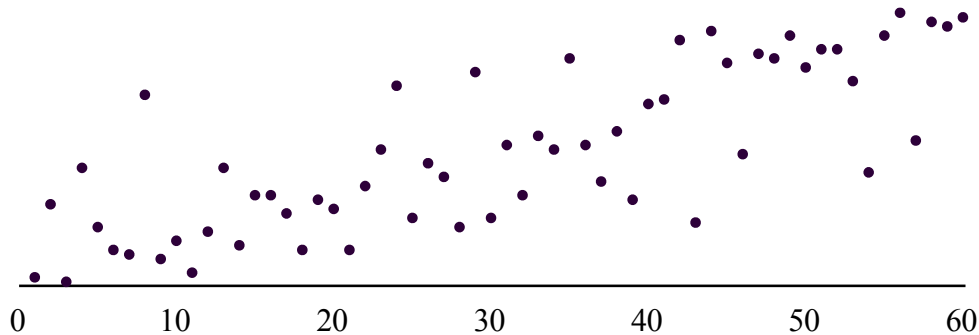
Answers—1; 2: 3, 4, 5: 6;/: 7;

ACT: Approaching the Math Test

[Math on actstudent.org](http://math.on.actstudent.org)

The Math test includes 60 multiple-choice questions. Standard-time students have 60 minutes. Most students find this section difficult to finish on time.

The most important characteristic of the Math test is that questions are arranged *roughly* in order of difficulty, as shown in this example scatterplot:



More of the easier questions are at the beginning, and more of the harder ones are at the end. Questions 1-10 are easy as a group; 11-20 are a little harder; and 51-60 will be almost all difficult questions. However, you might encounter an unexpectedly easier or tougher single question at any point in the test. And, of course, you might be very good at a topic they consider tougher (or not great at one they consider “easier”!).

Here’s the bottom line: *make sure you get to see all 60 questions!* It would be unfortunate if you got stuck spending time on a question you weren’t sure how to do and, as a consequence, never got to one that you’d get right if only you’d saved time for it! ACT Math timing strategy is about making sure you get to see all 60 questions so that you’re able to answer every question you know how to do, whether it’s #1, #60, or anywhere in between.

How should you do the ACT Math Test?

- **First ~45 minutes:** Work through the entire section. Answer questions you know you can get right. Circle and skip questions you think you could get, but not quite as easily. Skip and do nothing on questions on which you feel clueless.
- **The rest of the time:** Go back to questions that you circled and try to answer them. This is a great chance to use Smart Strategies (Guess-and-Check, Making Up Numbers, Process of Elimination). If you finish those, try ones you left blank.
- **3 minutes to go:** Fill in empty bubbles now. Don’t try to rush it with 10 seconds to go. Then go back to what you were doing.

SAT: Introduction to Math

Questions fall into one of four categories:

Heart of Algebra — 35%

The SAT strongly emphasizes what it calls *Heart of Algebra* — in other words, linear equations, linear inequalities, and functions, alone or in pairs. Ensure that you are an expert on everything about linear equations, linear inequalities, and linear functions — solving them, graphing them, their various forms (standard, point-slope, slope-intercept), and solving pairs of them using substitution and elimination.

Problem Solving and Data Analysis — 30%

Questions on the problem-solving side of the *Problem Solving and Data Analysis* category will focus on real-world situations involving percent change and ratios. More questions will involve data analysis — basically, beginning statistics including measures of central tendency, measures of spread, confidence intervals, and the various ways to describe relationships between variables.

Passport to Advanced Math — 25%

The *Passport to Advanced Math* category includes typical topics from late Algebra 1/beginning Algebra 2: exponents and radicals, solving quadratic equations, functions, rational expressions, and so on.

Additional Topics in Math — 10%

What *Additional Topics in Math* really means is geometry; why the section isn't called "Geometry," nobody knows. We'll discuss later which geometry relationships you really need to memorize, and which ones you can rely on the reference information for. This category also randomly includes operations with complex numbers ($a + bi$).

Desmos and Your Calculator

You are allowed to use your handheld calculator and/or the Desmos calculator that is built into the digital SAT. In my opinion, the Desmos calculator is easier to use in almost every way. Becoming skilled at using the Desmos calculator will give you a significant advantage on the Math section.

My [Desmos Practice for the Digital SAT](#) provides:

1. An extensive list of Desmos keyboard shortcuts.
2. Several practice questions (which we'll typically do together during a lesson) designed to demonstrate the many ways you can use Desmos to solve SAT questions.

Creating and Interpreting Linear Equations

[Worksheet](#), [Algebra Lab](#), [Saylor](#)

A few fundamentals make these questions easy.

1. First: notice what each variable stands for. (Or, sometimes, this is the question.)
2. Any value that repeats — cost per item, number of times per month, points per question, etc. — is the slope. It is always multiplied by the variable term.
3. Any one-time value — a setup fee, the amount of money you start with, the initial depth of a lake, etc. — is a y-intercept. It is a constant, disconnected from the variable.
4. If you're not counting something (like the first 3 minutes of a phone call are free) or you're counting something separately (like the first 3 minutes are at one price, and the rest are at a different price), subtract this from the variable term in parentheses.

Ex. A

A phone call costs 20 cents per minute. What is the cost, C , of an m -minute phone call, in cents?

Ex. C

A phone call costs 8 cents per minute after a free first minute. What is the cost, C , of an m -minute phone call, in cents?

Ex. B

A phone call costs 15 cents per minute plus a 10-cent connection fee. What is the cost, C , of an m -minute phone call, in cents?

Ex. D

A phone call's cost is a connection fee of 23 cents, which includes the first three minutes, and a fee of 7 cents per minute after that. What is the cost, C , of an m -minute phone call, in cents?

A. Since the cost is repeating (per minute), the cost is multiplied by the variable term. The answer is $C = 20m$.

B. The repeating part of the cost is multiplied by the variable term to get $15m$. The one-time cost, 10, is by itself. The answer is $C = 15m + 10$ cents.

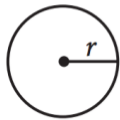
C. We aren't counting the first minute, so we subtract this from the variable term to get $(m - 1)$. Since 8 cents/minute is a repeating cost, it is multiplied by the variable term. The answer is $C = 8(m - 1)$.

D. 23 cents is the initial fee, so 23 stands alone. 7 cents/minute does not apply to the first three minutes, so we subtract 3 from the variable term to obtain $(m - 3)$. We multiply the repeating price, 7 cents per minute, by the variable term to obtain $7(m - 3)$. We then add the fixed cost. The answer is $C = 7(m - 3) + 23$.

Geometry Relationships

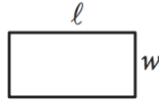
The figures below are from the SAT's reference information. Note that no reference information is available on the ACT.

- You must memorize this row of 2-dimensional (plane geometry) relationships:

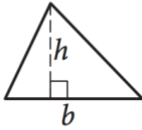


$$A = \pi r^2$$

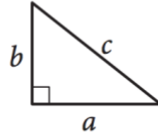
$$C = 2\pi r$$



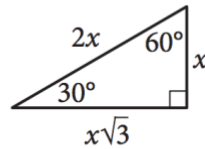
$$A = \ell w$$



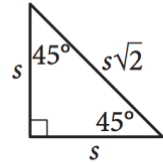
$$A = \frac{1}{2}bh$$



$$c^2 = a^2 + b^2$$



Special Right Triangles

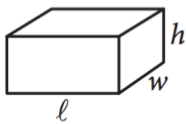


- You must also memorize the area and volume relationships for squares and cubes:

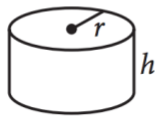
$$A = s^2$$

$$V = s^3$$

- Try to memorize these 3-dimensional relationships, but at least practice using them:



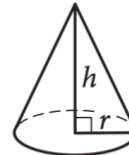
$$V = \ell wh$$



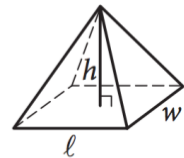
$$V = \pi r^2 h$$



$$V = \frac{4}{3}\pi r^3$$



$$V = \frac{1}{3}\pi r^2 h$$



$$V = \frac{1}{3}\ell wh$$

- This set should be memorized, too (*Note: $180^\circ = \pi$ is easier than $360^\circ = 2\pi$*)

The number of degrees of arc in a circle is 360.

The number of radians of arc in a circle is 2π .

The sum of the measures in degrees of the angles of a triangle is 180.

- Finally, memorize Pythagorean Triples, *sets of integers that fit into the Pythagorean Theorem*. The basic Pythagorean Triples are:

$$3, 4, 5$$

$$5, 12, 13$$

$$7, 24, 25$$

Multiples of these ratios work the same way. For example:

$$4 \times 3, 4, 5 = 12, 16, 20$$

$$2 \times 5, 12, 13 = 10, 24, 26$$

$$0.1 \times 7, 24, 25 = 0.7, 2.4, 2.5$$

Geometry Relationships Quiz — print this or do it on a separate piece of paperTwo-Dimensional Relationships

- Area of a Circle:
- Diameter of a Circle:
- Circumference of a Circle:
- Area of a Rectangle:
- Area of a Triangle:
- Pythagorean Theorem:
- Area of a Square:
- Volume of a Cube:
- Draw and label the two types of Special Right Triangles:

Three-Dimensional Relationships:

- Volume of a Rectangular Prism (Box):
- Volume of a Cylinder:
- Volume of a Sphere:
- Volume of a Cone:
- Volume of a Pyramid:

Other Relationships

- Degrees in a Circle:
- Relationship Between Degrees and Radians:

Pythagorean Triples (3 sets):

Smart Math Strategy: Guess-and-Check

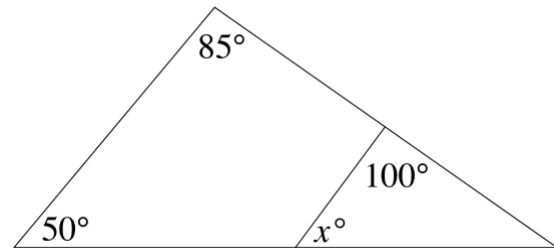
You can use *Guess-and-Check* when the question gives you the output or finishing value, and asks you for the input or starting value.

1. Joe starts with x dollars. He spends half, gets another \$10, and then spends half of what he has at that point. If this leaves him with \$8, what is x ?

- (A) 8
- (B) 10
- (C) 12
- (D) 18
- (E) 22

2. If the sum of twice x and the square of x is 63, which of the following could be x ?

- (A) 7
- (B) 8
- (C) 9
- (D) 10
- (E) 11



3. Given the diagram above, what is x ?

- (A) 15
- (B) 20
- (C) 25
- (D) 30
- (E) 35

4. Fred has \$1's, \$5's, and \$20's. He has fifteen bills and a total of \$119. If he has equal numbers of \$1's and \$20's, how many \$5's does he have?

- (A) 5
- (B) 7
- (C) 9
- (D) 11
- (E) 13

Steps for Guess-and-Check:

1. Identify the outcome you're looking for — for example, \$8 in #1, 63 in #2, or \$119 in #4.
2. On the ACT, start with answer (B)/(G) or (G)/(J) — whichever you think is more likely. On the SAT, start with answer (B) or (C) — whichever you think is more likely.
3. Input that value into the question and see if you get the outcome you identified in Step 1.
4. If yes — or if no, but you can determine the right answer with certainty — you're done.
5. If necessary, try the next useful answer choice. The pattern will tell you the answer.
6. Worst case scenario: just keep trying answers until one of them works!

You will rarely try anything other than the choices listed above to get the right answer!

Guess-and-Check PracticeAnswer explanations at baytutoring.com/resources

5. What is x if the average (arithmetic mean) of 3, 17, and x is 19?

- (A) 34
- (B) 35
- (C) 36
- (D) 37
- (E) 38

6. If $f(x) = x^2 + 3x - 8$, and $f(r) = 62$, what is r ?

- (A) 4
- (B) 5
- (C) 6
- (D) 7
- (E) 8

7. Given that $a = \frac{b}{3} + 1$, $b = \frac{c}{2} + 2$, and $a = 5$, what is c ?

- (A) 12
- (B) 14
- (C) 16
- (D) 18
- (E) 20

8. Charlie purchased 7 video games for a total cost of \$270. If all of the games cost either \$35 or \$40, how many \$40 video games did Charlie purchase?

- (A) 1
- (B) 2
- (C) 3
- (D) 4
- (E) 5

9. Sarah is 3 inches taller than Camille. Marlyn is 6 inches shorter than Louise and 5 inches shorter than Camille. If Louise is 68 inches tall, how tall is Sarah?

- (A) 69 inches
- (B) 70 inches
- (C) 71 inches
- (D) 72 inches
- (E) 73 inches

10. The average age of a group of four people is 49 years. If one of the people is 40 years old, what is the average age of the other three people?

- (A) 52
- (B) 54
- (C) 56
- (D) 58
- (E) 60

Note: the next 6 questions might require more thought about which answers to check.

11. The height in inches, h , of one plant d days from now can be represented as $h(d) = 4 + .07d$. The height of a second plant in inches, k , can be represented as $k(d) = \sqrt{d} + 1.12$. After how many days will the two plants be the same height?

- (A) 16
- (B) 18
- (C) 20
- (D) 22
- (E) 24

12. Given $f(x) = \frac{4x}{x^2-3}$, for what value of a is $f(a) = f(3)$?

- (A) -5
- (B) -4
- (C) -3
- (D) -2
- (E) -1

$$|2x - 5| > x + 3$$

13. Which of the following is NOT a solution to the equation above?

- (A) 0
- (B) 5
- (C) 10
- (D) 15
- (E) 20

$$\sqrt{x + 18} + 6 = x + 4$$

14. What is the solution set to the equation above?

- (A) $\{-2\}$
- (B) $\{2\}$
- (C) $\{7\}$
- (D) $\{-2, 7\}$
- (E) $\{2, 7\}$

$$\frac{-4 + a}{-4 - a} = -\frac{1}{3}$$

15. Given the equation above, which of the following could be a ?

- (A) 2
- (B) 4
- (C) 6
- (D) 7
- (E) 8

16. The perimeter of a rectangle is 24, and its area is greater than 30. If all of its sides are integer lengths, what is the greatest possible length of one of its sides?

- (A) 6
- (B) 7
- (C) 8
- (D) 9
- (E) 10

Guess-and-Check Post-Test

For this post-test, solve using only Guess-and-Check, even if you know another way.

1. During a certain week in Andiville, the high temperature on Tuesday was double the high on Monday. On Wednesday, the high was 4° greater than on Tuesday. On Thursday, the high was two-thirds of Wednesday's high temperature. If the high on Thursday was 12° , what was the high temperature on Monday?

- (A) 7°
- (B) 8°
- (C) 9°
- (D) 10°

2. The difference between a number and its square root is four more than half of the number. What is the number?

- (A) 4
- (B) 9
- (C) 16
- (D) 25
- (E) 36

3. Alfonso's lunches during a five-day workweek cost him \$9.25 on average. If his average lunch for the first three days of the week cost \$8.75, how much was his average lunch on Thursday and Friday?

- (A) \$9.50
- (B) \$9.75
- (C) \$10.00
- (D) \$10.25
- (E) \$10.50

4. Alfredo sells two kinds of cookies, one of which he sells for \$2 and the other for \$2.50. The number of the less-expensive cookies that he sells is 40% higher than the number of more expensive cookies. If he sells \$159 of cookies on a certain day, how many of the more-expensive cookies did he sell?

- (A) 15
- (B) 20
- (C) 25
- (D) 30

SAT Math Question Shortcuts
[Practice Questions on Resources Page](#)

What is the sum of the solutions of this quadratic equation?

The sum of the solutions to ANY quadratic equation in standard form is $-\frac{b}{a}$

What is ___ if the system of equations has no solutions/infinite solutions?

- If a system of equations has no solutions, use $\frac{a_1}{a_2} = \frac{b_1}{b_2}$.
- If a system of equations has infinite solutions, use $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

If two expressions are always equal/equal for all values of x (e.g., $ax^2 + bx + c = 4x^2 + 7x - 2$), then each of their parts is equal (e.g. $a = 4, b = 7, c = -2$).

Which of the following is equivalent to a rational expression (like $\frac{5x+2}{x-5}$)?

Use long division or Making Up Numbers.

For questions about the number or type of solutions to a quadratic equation...

Evaluate the discriminant (or graph with your calculator).

What is the minimum/maximum...

Generally, the minimum is the next integer *up*, and the maximum is the next integer *down*.*

Shortcutting percent questions:

Whatever you're comparing to (look for *than, of, compared to, etc.*) is on the bottom of the fraction. Percent increase must be > 1 , and percent decrease must be < 1 .

In a formula, if something happens to one variable, what happens to another?

If you can't do it analytically: get rid of numbers and variables that aren't involved. Make Up Numbers for the ones that are involved. Compare two results.

If two angles add up to 90° ...

The sine of one is the cosine of the other (and vice-versa); same thing with tangent and cotangent.

If you're asked to factor/find solutions for a cubic (or higher)...

You can factor by grouping if there are no other quick ways to do it.

ACT: Approaching the Reading Test

[Reading on actstudent.org](http://actstudent.org)

The Reading test includes four reading passages with ten questions each. Standard-time students have 35 minutes for the Reading test, and most students find this section difficult to finish. To finish on time, a student would need to average about 8½ minutes per passage. You will always get one each of four types of passages:

- Prose Fiction
- Humanities
- Social Science
- Natural Science

The passage types will be labeled, and you should always start with the passage types with which you're most comfortable. There's no reason or need to go in order.

Time Strategy (for standard-time students)

Through practice, learn how long it takes you to do ACT reading passages. It is better to do 3 passages well than to do 4 of them hurriedly. Therefore, if you discover that you need more than 9 minutes to do a typical ACT reading passage, focus on doing a great job with three of the passages and stick with guessing and a few rushed answers for the fourth.

How should you do ACT Reading passages?

Very few reading questions on the ACT provide line numbers for reference. Additionally, questions are not asked in order of how the ideas appear in the passage.

Therefore, it is usually not practical to try to answer questions as you go along. Your steps for reading ACT passages and answering questions are as follows:

1. Save the paired passage for last. If you get in a time crunch, that's the best passage to have remaining because you can quickly read and answer questions from one of the passages. In that case, choose the passage about which more questions are asked.
2. When you get to each passage, read any introductory material. Then read the passage for general meaning and structure. As you read, **annotate each paragraph with a 1-2 word summary**. Think of the Cornell note-taking system; this will serve as a guide or index to help you find information as you answer questions.
3. Answer the questions in any order — they aren't in chronological order anyway.
4. The information you need to answer specific questions is often 2-4 lines before or after any line numbers given to you. More generally, it's not always exactly where it seems like it would be. The ACT will reward you for being observant and thorough, and not just looking at one line and assuming it's enough. But on the ACT, *the answer is almost always directly stated in the passage. Find it!*

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ACT: Approaching the Science Test

[Science on actstudent.org](http://actstudent.org)

The ACT Science test does not test science. It does not require you to know chemistry, biology, physics, geology, or any other scientific discipline. The Science test measures your ability to think and reason scientifically. Standard-time students have 35 minutes to do 40 questions, just like the Reading test, but this test tends to go slightly quicker.

Science passages are purposefully crafted around material that test takers are unlikely to have seen before. If the material were familiar to students, it would be a test of scientific knowledge. Because ACT Science material is unfamiliar to all students, everyone must rely on scientific reasoning and logic skills, which is what the Science test intends to measure.

There are three passage types:

- **Data Representation** (2 passages, 6 questions each):
Read data points from charts, graphs, and pictures, and make interpretations.
- **Research Summaries** (a.k.a. Experiments; 3 passages, 7 questions each):
Understand descriptions and results of multi-step experiments, and make inferences.
- **Conflicting Viewpoints** (1 passage, 7 questions):
Short paired reading passages with compare/contrast questions.

The Science test is very similar to the Reading test. You're asked to sort through a large amount of information — too much to remember as you go along — that is only somewhat related to information you've seen before, find specific and general information, and make inferences. As a result, **ACT reading strategies are just as effective on the Science Test!**

- Develop your own answer first.
- The information is always in the passage.
- Use process of elimination.
- Do passage types that you're most comfortable with first.

How should you do ACT Science passages?

1. For most students, do not read the text of the passage. You will go back to it for any information that you need. (Note: DO read Conflicting Viewpoints passages. Answer questions about each viewpoint after reading it.)
2. DO carefully determine the *format* of the figures and tables. Check the units, scales, keys, etc.: "**Look at everything outside before anything inside.**"
3. Refer to the Figures and Tables referenced in each question to get your answers.
4. On confusing questions, look for key words, numbers, and units on graphs/charts.
5. Only go back to the passage to search for specific definitions or other information necessary to answer a particular question.

UNIT 2

Pronouns

[CCC 1](#), [CCC 2](#), [CCC 3](#), [CCC 4](#), [IXL \(filter by 11th\)](#), [Chomp 1](#), [Chomp 2](#)

Pronouns take the place of specific nouns.

- Common **pronouns** include *he, she, him, her, they, them, and it*.
- Common **possessive pronouns** include *his, her, hers, their, theirs, our, ours, and its*.

To evaluate a pronoun, look at two elements of its usage. First is **antecedent reference**: a pronoun must refer with *absolute clarity* to the right noun.

1. Sal and Wilson walked towards me, almost threateningly, before he finally smiled, laughed, and said hello.
2. As a result of their declining value, stores are selling televisions at very low prices.
3. A big city sometimes appears to have grown without any forethought whatsoever, but they actually spend a lot of time planning the smallest details of neighborhoods years or decades into the future.
4. The projects are designed to assess students' comprehension of important topics, but whether it actually meets the required district standards is uncertain.
5. The investment advisor's first five new clients all came from the referrals of one generous business partner and their friends.
6. The potholes have become a danger, and they have not yet proposed a reasonable solution.

Second is **antecedent agreement**: nouns are singular or plural, as are pronouns; in any pronoun usage, the two must match.

7. I saw someone try to jump Springfield Gorge, but they didn't make it.
8. The Jonas Brothers posters arrived on time, but we didn't put them up for another month.
9. Not only is the city of San Diego a beautiful place to live, but because the folks who live there get to spend so much time at the beach, its people all seem to be tanned and beautiful as well.
10. When it came time for her big game, Jane's team realized that effort could only carry a team so far; talent was also a prerequisite to success.
11. The eighth-grade class, having waited all day for an assembly that was supposed to be entertaining, found their time instead wasted by a school spirit rally that evinced little if any positive emotion at all.

Pronouns Post-Test

Every dog has **1** its day, but nobody

knows when **2** their day will come. That's the reality of life: good things will happen

— as will bad ones — but when **3** it will happen, nobody knows. That doesn't mean that we operate blindly. Rather, it means that we go along as best we can, doing what we believe to be proper at any given interlude. **4** They are both realistic and optimistic. They are prepared and

positive: when **5** its time for them to take action, these people are ready. And when

fate intervenes, **6** who's fault is that? It is

not the fault of a single person; **7** he simply understands how the world works. The uncertainty of life is not something that goes away, nor is the excitement of seeing what's around the corner. In fact, far from being a point of anxiety, **8** it is in fact the best part of life.

1. (A) its
(B) it's
(C) its'
2. (A) their
(B) they're
(C) his or her
3. Which is NOT acceptable?
(A) it
(B) they
(C) those
4. (A) They are
(B) It is
(C) Those who operate this way are
5. (A) its
(B) it's
(C) its'
6. (A) who's
(B) whose
(C) whos
7. (A) he simply understands
(B) they simply understand
(C) such a person simply understands
8. (A) it
(B) this
(C) this excitement

Punctuation: Apostrophes, Commas, and Dashes

[Chomp commas \(scroll down\)](#), [Chomp PowerPoint](#), [Chomp](#), [IXL \(J, K, L\) CCC](#)

Apostrophes

Apostrophes indicate possession, and they go after whoever the owner is. (Apostrophes are *never* be used to pluralize, ever.) *To use an apostrophe, write the owner, then put an apostrophe. Stated another way: whatever is in front of the apostrophe must be the owner:*

- Roommates means several of these people: “I have 2 roommates.”
- Roommate’s means belonging to roommate: “my roommate’s bedroom.”
- Roommates’ means belonging to roommates: “the roommates’ group decision.”

Commas and Dashes

1. Use a comma to separate a dependent clause from the complete sentence before or after.
2. Descriptive/parenthetical vs. identifying clauses:
 - Use commas to set off a *descriptive/parenthetical* clause from the sentence around it.
 - Do NOT use a comma around an *identifying* clause/phrase.
 - The years 2003-2005, *when I lived in China*, were interesting years in my life. (Even without this phrase, we would still know that we're talking about 2003-2005. Therefore it is merely *descriptive*, so commas are used.)
 - The time *when I lived in China* was an interesting time in my life. (Without this phrase, we would not know which time we're talking about. Therefore it is not just descriptive but *identifying*, so no commas are used.)
3. Know when to use a comma with and/but/or:
 - Joining two sentences = comma.
 - Joining any two units *except* complete sentences = no comma!
 - Joining three or more items in a list = commas between them, but not before.
4. Use commas correctly with a series of adjectives:
 - Interchangeable = commas
 - Not interchangeable = no commas
 - The flaky, buttery croissant was the perfect breakfast treat.
 - The steaming scrambled eggs and crispy organic bacon looked good too.
5. One dash replaces a comma *following a complete sentence only*.
 - You should learn dash rules — and if you do, you'll get more questions right.
6. Two dashes surround parenthetical clauses, just as commas or parentheses would.
 - Learning dash rules (as you're doing right now) is a great idea.
 - Learning dash rules, as you're doing right now, is a great idea.
 - Learning dash rules — as you're doing right now — is a great idea.

Punctuation Post-Test

Mexican cuisine is not **1** defined, solely by the presence of stereotypical

2 ingredients: guacamole, sour cream, cheese, and so on. Rather, it is stunningly

3 varied there are many ingredients that an American would simply not expect to see, like squash blossoms. Furthermore, even talking about "Mexican food" is

misleading **4** as, the cuisine is highly regionalized. In fact, if you asked them to describe the cuisine of their country,

5 Mexican's responses would likely vary

considerably. Some would **6** say: that their cuisine revolved around seafood.

Others would talk about **7** vegetables; and

still others about **8** spice's.

1. (A) defined, solely
(B) defined, solely,
(C) defined solely
2. (A) ingredients:
(B) ingredients;
(C) ingredients,
3. Which is NOT acceptable?
(A) varied —
(B) varied:
(C) varied;
4. (A) as, the
(B) as the
(C) as: the
5. (A) Mexican's
(B) Mexicans'
(C) Mexicans
6. (A) say: that
(B) say that
(C) say that,
7. (A) vegetables; and
(B) vegetables. And
(C) vegetables, and
8. (A) spice's
(B) spices'
(C) spices

Subject-Verb Agreement

[CCC](#), [IXL](#), [ChompChomp](#)

Removing prepositional phrases, descriptive clauses, and other connected independent and dependent clauses allows you to more easily evaluate a single subject-verb agreement pair.

Removing Prepositional Phrases

1. The cat on the table is cute.
2. The washing machines in the laundromat under my apartment in the city is not well maintained.
3. The lady with those other people was wearing a beautiful dress.
4. A chicken in every pot make for a shortage of empty pots.
5. The parks between the museums near downtown is finally finished being renovated.
6. An answer beyond our understanding is still an answer.

Removing Descriptive Clauses (often start with -ing words, of, that, than, w-words, etc.)

7. The moment when I got my first kisses are still crystal-clear in my mind.
8. Folks who think that they are better than other people are usually wrong.
9. The owner of three of the area's most successful businesses actually comment on issues only after plenty of thought.
10. The guy running around the lake in yellow short-shorts is a friend of mine.
11. Surfers riding the waves off the coast reminds me of home.
12. Not one of the fishes in the tank are anything but beautiful.

Removing Connected Independent and Dependent Clauses

13. I said they might be here, and sure enough one of the men whom I mentioned to you is standing over there now.
14. My dogs might be old, but considering their ages, they are remarkably energetic.
15. Nobody, even those who knew that an announcement was imminent, were expecting such exciting news.
16. John's car, despite being older than most cars on the roads, are still quick.

Verb Agreement With Multiple Subjects: the subject is the one closer to the verb

17. Either Michael or Alonzo is stopping by at 5:00 to say hello.
18. We will determine what people or group are responsible for this act.

More with Verbs

[CCC 1](#), [CCC 2](#), [CCC 3](#), [PowerPoint](#), [IXL](#)

Subject/Verb Agreement

Underline the subject, then circle the correct verb.

1. The idea that some people shouldn't have the same rights as others (is/are) unjust.
2. One of the best baseball players in any of the six leagues (is/are) surprisingly still not going to the All-Star Game.
3. Even after hours and hours of study, the piles of data generated from the recent experiment (was/were) still confusing to me.
4. After my experience trying to arrange a test drive with a rude salesperson, I wondered if even one of the folks from those car dealerships (sell/sells) more than a couple of cars each year!
5. Joe read several articles that (relate/relates) company size to employee happiness.
6. The fact that I feel so much love and appreciation from all of my friends (carry/carries) me through difficult days.

Verb Tense

The sentence will always include a context to determine past, present, future, etc.

7. Jane came home to find that her most precious belongings (had/have) been stolen.
8. It (was/is) with pleasure that the host graciously showed her guest to the living room.
9. After a wonderful day at the beach, I realized that I (had been carrying/am carrying) a great deal of sand in my shoes.
10. Nobody gave the Tigers a chance to win, but now with the score tied at halftime there (remained/remains) a chance that they could pull off the upset.
11. (Living/Having lived) in Montana for twelve years, Frederick could hardly remember the sound of the ocean upon his return to California.
12. When the tour ends in three weeks, future opportunities for the crew's best dancer (will rest/is resting/are resting) in the hands of the crew's management, sponsors, choreographers, among others.

Verbs Post-Test

Nobody in the room **1** is more knowledgeable than Professor Sally Lexington. This is what all of the audible

chatter of the conference attendees **2** tell me, yet I find this strange. Why? Because Professor Lexington isn't speaking, nor

will she **3** be. Rather, I spot her sitting alone near the back of the conference

room. The back **4** rows of a conference room is no place for one of the brightest minds in the field, I reasoned, so I cautiously strode her way.

Before I had taken even three steps her way, Professor Lexington's smile beamed brightly in my direction and **5** beckoned me to engage with her. The woman whom I had often wondered about and

6 admired above all of

7 her academic peers was now my partner in conversation.

8 The pleasures of engaging in this discussion surely was all mine, but she was indulgent beyond kindness.

1. (A) is
(B) are
(C) was
2. (A) tell me,
(B) are telling me,
(C) tells me,
3. (A) be.
(B) being.
(C) speaking.
4. Which is NOT acceptable?
(A) of a conference room is
(B) row of a conference room is
(C) row of a conference room being
5. (A) beckoned me to
(B) beckons me to
(C) beckon me to
6. (A) admired
(B) admires
(C) is admiring
7. (A) her academic peers was
(B) her academic peers were
(C) her academic peer was
8. (A) The pleasures of engaging in this discussion surely was

(B) The pleasure of engaging in this discussion surely were

(C) The pleasures of engaging in this discussion surely were

Parallel Structure

[CCC 1](#), [CCC 2](#), [CCC 3](#), [IXL 1](#), [IXL 2](#), [Chomp Chomp](#)

Parallel structure means that different parts of a sentence have equivalent grammatical forms.

Not parallel: I like to eat, drink, and playing cards.
 Parallel: I like to eat, drink, and play cards.
 Or: I like eating, drinking, and playing cards.

Find mistakes and make corrections where necessary.

- | | |
|---|---|
| <p>1. The football team was known for its bruising offense, fabulous defense, and it had a great coaching staff.</p> | <p>5. The flower arrangement was both delicate in its construction yet somehow sturdiness in its appearance.</p> |
| <p>2. As the lead singer fumbled his solo, everyone in the crowd seemed to alternately look away in horror and staring in astonishment at the cringe-inducing spectacle.</p> | <p>6. The rhetoric used by those in favor of the change was both brimming with passion and grounded in reason.</p> |
| <p>3. While Martin didn't mind doing housekeeping from time to time, he did mind to be taken advantage of by those who thought he would be a pushover about it.</p> | <p>7. The fear of being caught unprepared was equally motivating to Phil as was the desire to feel that he was ready for any and all eventualities.</p> |
| <p>4. Dezi has advocated for the community for years by speaking about its needs and maintaining her view that others should do the same.</p> | <p>8. The researchers were enthusiastic about the subjects' performance in the initial trials and other promising developments and had optimism about where their work might lead.</p> |

Parallel Structure Post-Test

What should kids learn in school? Math teaches students systematic thinking; science, critical reasoning; and history, **1** to understand cause-and-effect relationships. All of these subjects have

their place in school curricula, **2** while other subjects receive short shrift. Many

educators argue—and many **3** parents are agreeing—that the arts are undervalued in their ability to help children develop. Yet as they advocate for the least "scholarly" of subjects, I would like to

4 advocate for it which is the most: the study of formal logic. Classes in logic teach students to think, reason, and understand just as math, science, and history **5** enable students to individually.

The study of logic is **6** both less dry than it sounds and more valuable than the vast majority of people realize.

1. (A) to understand
(B) they understand
(C) understand
2. Which is NOT acceptable?
(A) while other subjects
(B) but other subjects
(C) while others are receiving
3. (A) are agreeing
(B) are in agreement
(C) agree
4. (A) advocate for it which is the most:
(B) do the same for the most
(C) be advocating for the most
5. (A) enable students to
(B) do
(C) are
6. (A) both less
(B) less both
(C) both

English/Writing and Language: Specificity and Repetition

Answer exactly what a specific question asks you!

Anytime there's an actual question in the Writing and Language section, instead of just an underline and some answer choices, focus intensely on *exactly* what the question is asking you for. These questions don't just ask you "which is best;" instead they give you extremely specific clues, so use them! Some example questions are:

Which of these following versions of the sentence best extends the claim the author made in the previous sentence?

Which of the following word choices best indicates the author's disappointment while maintaining the tone of the passage?

Repetitiveness and Wordiness

Purdue OWL ([1](#), [2](#), [3](#)), [Douglas](#), [CCC](#)

Partly it's just good English, and partly it's easily testable: don't say in 5 words what you could say in 3. Repetitive phrasing will be wrong, and extra words that do not add significant information will be wrong. Look out in particular for:

- Overdone description: "*The jungle was verdant, lush, and filled with plants.*"
- Subtle re-wording: "*The intense light was blinding and made it hard to see.*"
- Annual, monthly, etc.: "*The machine's hourly production is 4 units per hour.*"

Smart Math Strategy: Making Up Numbers

Use *Making Up Numbers* when the question is difficult because it's written in terms of variables, where those variables could have many different possible values.

Ex. A

What is the average of $x + 3$, $5x - 4$, and $3x + 7$?

- (A) $3x + 2$
- (B) $3x + 3$
- (C) $5x$
- (D) $5x + 2$

Ex. B

If $a = 3b$ and $b = 4c$, what is c in terms of a ?

- (A) $\frac{a}{12}$
- (B) $\frac{a}{7}$
- (C) $\frac{3a}{4}$
- (D) $\frac{4a}{3}$

Ex. C

If $b - a = 5$, what is the distance in the standard coordinate plane between the points $(a, 2b)$ and $(b, 2a)$?

- (A) $5\sqrt{2}$
- (B) $5\sqrt{3}$
- (C) 10
- (D) $5\sqrt{5}$

Ex. D

At a certain restaurant, the first pizza costs d dollars and every additional pizza costs x dollars. What is the average price per pizza when y pizzas are purchased?

- (A) $\frac{xd}{y}$
- (B) $\frac{(d + xy)}{y}$
- (C) $\frac{(d + xy - x)}{y}$
- (D) $\frac{y(x + 2)}{2}$

Steps for Making Up Numbers :

1. Choose a different number for each variable in the question.
 - Use low positive numbers or numbers that fit the question, like 100 for a percent.
 - Do not choose 0 or 1, or any numbers that could make things complicated.
2. Replace *every* variable with numbers, in both the question and all of the answer choices.
3. Solve the resulting arithmetic question. (And/or, use process of elimination to get rid of unreasonable answers.)
4. If two or more answer choices yield the correct value, change your numbers.

Smart Math Strategy: Making Up NumbersExplanations at baytutoring.com/resources

1. If $x - 1$ is a multiple of 3, which of the following must be the *next* multiple of 3?

- (A) $x - 4$
- (B) $x + 2$
- (C) $2x - 2$
- (D) $3x - 3$

2. If $a = 3b$ and $b = 4c - 1$, then what is $a + 3$ in terms of c ?

- (A) $12bc - 3b$
- (B) $3b + 4c + 2$
- (C) $12c - 3$
- (D) $12c$

3. A machine makes r objects in s minutes. Which of the following represents the number of objects made in 3 hours?

- (A) $3rs$
- (B) $\frac{3s}{r}$
- (C) $\frac{180r}{s}$
- (D) $\frac{180s}{r}$

4. A certain bus stops every m minutes on average. If a bus travels for h hours, on average how many stops would it make?

- (A) $\frac{h}{m}$
- (B) $\frac{60h}{m}$
- (C) $\frac{h}{60m}$
- (D) $\frac{60m}{h}$

5. Which of the following expressions represents the maximum number of regions into which a rectangle can be divided by x horizontal lines and y vertical lines?

- (A) xy
- (B) $x + y + 2$
- (C) $2xy$
- (D) $(x + 1)(y + 1)$

6. A phone call is charged at c cents, which covers up to 3 minutes, and then x cents per minute after the first three. What is the total cost in cents of an m -minute phone call?

- (A) $c + mx$
- (B) $mx - c$
- (C) $c + x(m - 3)$
- (D) $c + m(x - 3)$

7. If $a < b < -1$, which of the following must also be true?

- (A) $a + b > a - b$
- (B) $a^2 > b^2$
- (C) $2a > 2b$
- (D) $-\frac{a}{2} < -\frac{b}{2}$

8. A square has sides of length x , and a rectangle has width y and a length twice that of the square. If the rectangle has a larger area, what is the difference in area between the rectangle and the square?

- (A) $2y - x$
- (B) $xy - x^2$
- (C) $x(2y - x)$
- (D) $y(2x - y)$

9. A is the average of a group of n items, and one item in the group has a value of k . What is the new average when the item with value k is removed?

- (A) $A - \frac{k}{n}$
- (B) $\frac{An-k}{n}$
- (C) $\frac{A(n-k)}{n}$
- (D) $\frac{An-k}{n-1}$

10. A group of a items has an average of p , and a group of $2a$ items has an average of r . What is the overall average when the two groups are combined?

- (A) $\frac{(p+r)}{2}$
- (B) $\frac{(p+2r)}{3}$
- (C) $\frac{(2p+r)}{3}$
- (D) $\frac{(ap+2ar)}{3}$

Making Up Numbers Post-Test

For this post-test, solve using only Making Up Numbers, even if you know another way.

1. Given that X and Y are digits, which of the following represents the product of $0.Y$ and $0.0X$?

(A) $X \cdot Y$

(B) $\frac{X \cdot Y}{10}$

(C) $\frac{X \cdot Y}{100}$

(D) $\frac{X \cdot Y}{1,000}$

(E) $\frac{X \cdot Y}{10,000}$

2. The average of two numbers, t and v , is a . Which of the following correctly expresses t in terms of the other two variables?

(A) $t = \frac{1}{2}a - v$

(B) $t = \frac{1}{2}a + v$

(C) $t = a - v$

(D) $t = 2a + v$

(E) $t = 2a - v$

3. A process that was supposed to begin at 1:00 PM instead began 5 minutes late. If the process takes m minutes to complete, how many minutes remain in the process when it is p minutes past 1:00 PM?

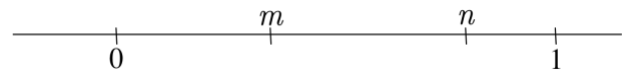
(A) $m + p + 5$

(B) $m + p - 5$

(C) $m - p + 5$

(D) $m - p - 5$

(E) $p - m + 5$



4. The numbers m and n are shown on the number line above. Which of the following lists is correctly ordered from least to greatest value?

(A) $mn, \frac{m}{n}, \frac{1}{m}$

(B) $mn, \frac{1}{m}, \frac{m}{n}$

(C) $\frac{m}{n}, mn, \frac{1}{m}$

(D) $\frac{m}{n}, \frac{1}{m}, mn$

(E) $\frac{1}{m}, mn, \frac{m}{n}$

Definitions You Should Know

Integers are like whole numbers but include negatives and zero. Numbers like 6, 0, -8, and 1,422 are integers. Numbers like 0.6, $\sqrt{3}$, and $7\frac{3}{5}$ are not. **Whole numbers** are integers starting from 0 (Zero looks like a "hole"). **Natural numbers** are integers starting at 1.

The **mean** or **arithmetic mean** of a group of numbers is just the average of those numbers. The terms mean, arithmetic mean, and average all mean the same thing. They are calculated by adding up all of the items and dividing by the number of items.

The **median** of a group of numbers is the middle number *when the numbers are listed in order from least to greatest*. Another definition for median is that *there are as many items above as there are below*.

The **mode** of a group of numbers is the most common number. The mode of 2, 2, 3, 4, 4, 4, 5 is 4, because it occurs the most. The list 6, 6, 7, 7, 7, 8, 9, 9, 9 has two modes, 7 and 9.

A **digit** is one of the following: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. These are the only digits. Know place names, too: going left from the decimal point: units (ones), tens, hundreds, etc. Going left from the decimal point: tenths, hundredths, thousandths, etc.

Factors are terms that multiply to make up a bigger number. The factors of 28 are 1, 2, 4, 7, 14, and 28; the *prime factors* are 2, 2, and 7; the distinct prime factors are 2 and 7.

Prime numbers are numbers with *exactly* two factors: 1 and itself. Therefore 2, 3, 5, 7, 11, and many other numbers are prime. The number 1 is not prime. It only has one factor.

Multiples are what you get when you multiply a number by 2, 3, 4, etc. The first few multiples of 12 are 24, 36, 48, and 60. Multiples are bigger, factors are smaller.

The **Least Common Multiple (LCM)** of a set of terms is the expression that accounts for all of the factors of each term with no additional factors included. The LCM is all of the various factors to the highest power they are found in any term.

The **Greatest Common Factor (GCF or GCD, for Divisor)** of a set of terms is the number or expression composed only of the factors that all of the terms have in common. The GCF is all of the various factors to the lowest power they are found in any term.

Inclusive means including both endpoints. If you are counting integers from 3 to 6 inclusive, you'd include both 3 and 6. You'd count 3, 4, 5, and 6, for a total of four. (See "Counting" towards the end of this binder.)

X in terms of Y means you're solving for X. Your result looks like: $X = (\text{something with } Y)$.

Solving 2-by-2 Systems of Linear Equations

[Elimination](#), [Substitution](#), [Word Problems](#)

A 2-by-2 system of equations means 2 variables and 2 equations. There are two ways to solve such systems.

Ex. A:

$$\begin{aligned} 3x + 4y &= 15 \\ x + 7y &= 22 \end{aligned}$$

Substitution

Solve the 2nd equation for x in terms of y :

$$\begin{aligned} x + 7y &= 22 \\ x &= 22 - 7y \end{aligned}$$

Substitute this value into the 1st equation:

$$\begin{aligned} 3(22 - 7y) + 4y &= 15 \\ -17y &= -51 \\ y &= 3 \end{aligned}$$

Use this value in one of the original equations:

$$\begin{aligned} 3x + 4(3) &= 15 \\ x &= 1 \end{aligned}$$

Elimination

Multiply the 2nd equation by 3 to line up the x -terms:

$$\begin{aligned} 3(x + 7y = 22) &\text{ to obtain} \\ 3x + 21y &= 66 \end{aligned}$$

Subtract the equations from each other:

$$\begin{array}{r} 3x + 4y = 15 \\ -(3x + 21y = 66) \\ \hline -17y = -51 \\ y = 3 \end{array}$$

Use this value in one of the original equations:

$$\begin{aligned} 3x + 4(3) &= 15 \\ x &= 1 \end{aligned}$$

The solution is (1, 3).

1. $\begin{aligned} x + 4y &= 13 \\ 3x - 2y &= -17 \end{aligned}$

3. $\begin{aligned} 3y - 4x &= -34 \\ 7y + 2x &= 0 \end{aligned}$

2. $\begin{aligned} 2x - 7y &= -58 \\ 11x + y &= -3 \end{aligned}$

4. $\begin{aligned} 2x - 3y &= -4 \\ 4x - 5y &= -2 \end{aligned}$

Rates

Rate and *ratio* both come from the same root word, and they both indicate *a comparison made by division*. You should leave this page understanding the relationships between equations with two variables, graphs, slopes, rates, and ratios.

One defining characteristic of rates and ratios that should sound familiar from a few pages ago is that they have *units*. For example:

- A speed can be measured in miles per hour — or meters per second.
- Earnings can be measured in dollars per hour — or dollars per year.
- The growth of a sports team's fan base can be measured in people per game — or thousands of people per season.

The slope of a graph is its rate of change, with units of y/x , or in spoken terms, *y per x*. Compare the formulas using speed as an example rate. Rates and slopes are the same thing!

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{speed} = \frac{\text{position}_f - \text{position}_i}{\text{time}_f - \text{time}_i}$$

1. Bob's investment increased from \$14,000 to \$18,800 from January 1, 2015 to May 1, 2016. Determine the monthly rate of change, with units.

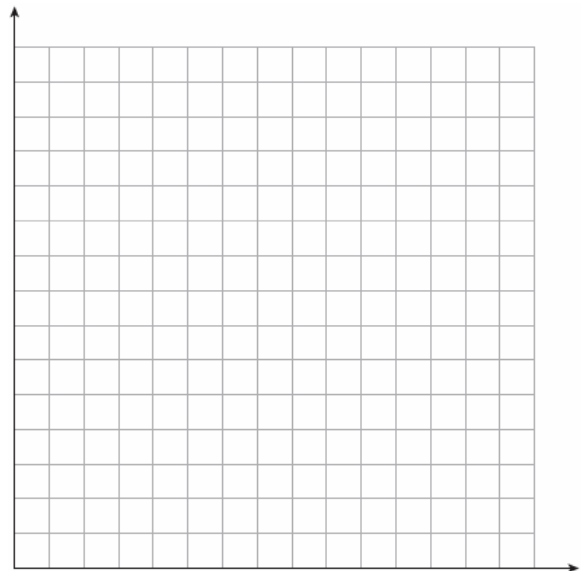
6. Assuming the rate of change remains constant, what will be the value of his investment in October 2016?

2. What is the slope of the graph of Bob's investment value over time, with units?

3. What is the y-intercept, with units, of the graph of Bob's investment value

4. Write a function $v(m)$ expressing Bob's investment value in terms of the number of months m after January 2015.

5. In what month was his investment valued at \$16,100?



7. Graph $v(m)$, noting appropriate units, in the space above.

Linear Equations and Inequalities Post-Test

A certain plant first appears above the surface of the soil 15 days after the seed is planted. From that point on, it grows by 0.22 cm per day.

1. Write a function, $h(d)$, that expresses the height of the plant, h , in centimeters as a function of the number of days, d , since its seed was planted.

2. How tall will the plant be after 50 days?

3. At the end of how many days after planting the seed will the plant be 1 meter tall?

4. The cost in dollars, C , of a pizza with a certain number of toppings is $C = 9 + 1.25(x - 1)$, assuming that at least one topping is selected. Select all of the true statements:

- I. A pizza with no toppings is \$9.00
- II. Toppings are \$1.25 each.
- III. Unlimited toppings are \$9.00.

5. Write and solve an equation for the number of toppings a person with \$14 could get on a single pizza.

6. The depth of a pool is 8'8", and it is being drained at a rate of 9" per hour. Write an equation for its depth in feet, d , in terms of the number of hours after draining began, h .

7. Three sodas and two hot dogs cost \$15.50. Four hot dogs and five sodas cost \$28.50. How much are hot dogs and sodas individually?

8. A coffee shop pays \$600 each day in salaries, rent, and other fixed costs. Each cup of coffee they make costs them \$0.50 and sells for \$2.00. Each day, they give away three cups of coffee for free. Write an equation to determine the number of cups of coffee, c , the shop must *sell* to break even; then solve the equation.

Ratios and Proportions

[MathIsFun](#), [PurpleMath](#), [IXL \(basic\)](#), [Online](#), [Worksheet](#)

A proportion looks like:

$$\frac{a}{b} = \frac{c}{d}$$

It indicates that the *ratios* on both sides are equal. For example, these ratios are both 0.4:

$$\frac{2}{5} = \frac{14}{35}$$

Proportion questions on the SAT generally involve units, not just numbers. **The key to proportion questions is ensuring that the units are the same on both sides.** Example:

Ex. A: On a large map, 1 inch represents 2.5 miles. How many miles are represented by a distance of 1.3 feet on the map?

Step 1: Determine the units for the top and bottom of both fractions and *write out that fraction*. In this case, the fraction would look like:

$$\frac{\text{map inches}}{\text{real miles}}$$

Step 2: Write your proportion based on the “guide” that you just wrote:

$$\frac{\text{map inches}}{\text{real miles}} \quad \frac{1}{2.5} = \frac{15.6}{x}$$

You must convert 1.3 feet to 15.6 inches because the units must be equal on both sides! Cross-multiply and solve; $x = 39$.

Combined Part/Whole Questions

Here is an example of using different units to set up a complex ratio more easily.

Ex. B: A certain chemical mixture is 3 parts of Chemical A for every 5 parts of Chemical B. How many gallons of Chemical A are in 10 total gallons of the mixture?

$$\frac{\text{Amount of A}}{\text{Total Mixture}} \quad \frac{3}{8} = \frac{x}{10}$$

If you use these units, you must add 3 (parts of A) and 5 (parts of B) to get 8 units of Total Mixture. Cross-multiply and solve for $x = 3.75$.

Ratios and Proportions Practice

1. Janet can type 45 words in 40 seconds. How many words can she type in four minutes?
2. It takes four weeks to manufacture seven printing machines. How many machines could be manufactured in 12 days?
3. Elsa is a consultant who earns \$120 per hour actually spent with a client. She figures that each hour with a client requires 40 minutes of prep time. On average, how much money does Elsa make in a 40-hour workweek?
4. If a baseball team wins 11 games for every 7 it loses, how many games would it win in a 162-game season?
5. A batch of cookies has a ratio of 5 chocolate chip cookies to 7 peanut butter cookies. How many chocolate chip cookies are in a batch of 60 cookies?
6. If $8x = 12y$, what is the ratio of $2y$ to x ?
(A) 1:3 (B) 2:3 (C) 3:4
(D) 9:8 (E) 4:3
7. If the ratio of $4a$ to $3b$ is 7:5, what is the ratio of b to a ?
8. One-quarter of an ounce of gold is worth \$330, how much is seven-eighths of an ounce worth?
9. Yolanda finds that the number of profiles she likes on Match.com is directly proportional to the number of profiles she views. Yesterday, she looked at 30 profiles and liked 8 of them. If she wants to find 20 profiles she likes, how many should she expect to view?
10. A cookie recipe calls for a 2 to 3 ratio of flour to sugar. Every cup of the flour-sugar mixture makes a dozen cookies. How many cups of sugar are needed to make 4 dozen cookies?

Ratios and Proportions Post-Test

1. In a MunneyHunney candy bar, $\frac{4}{9}$ of the calories come from fat. If the candy bar has 450 calories, how many of the calories do NOT come from fat?

3. George's group of close friends includes 6 athletes and 2 non-athletes. If his friend group is representative of his school as a whole, how many athletes are there in his school of 1,440 students?

2. A certain manufacturing process can create 5 machines takes 14 hours. How many machines can it create in 7 days of non-stop operation?

4. For the season, a football team's average offensive drive has been 6.5 plays, of which 4.5 are running plays and the rest are passing plays. In a game where the team runs 52 total offensive plays, how many of them are likely to be passing plays?

Statistics: Measures of Central Tendency and Spread

[MathIsFun](#), [IXL 1](#), [IXL 2](#)

Measures of Central Tendency characterize the "middle" of a data set.

- The **mean** or **arithmetic mean** of a group of numbers is the average of those numbers. The terms mean, arithmetic mean, and average all mean the same thing. They are calculated by adding up all of the items and dividing by the number of items.
- The **median** of a group of numbers is the middle number *when the numbers are listed in order from least to greatest*. Another definition for median is that *there are as many items above as there are below*.
- The **mode** of a group of numbers is the most common number. The mode of 2, 2, 3, 4, 4, 4, 5 is 4, because it occurs the most. The list 6, 6, 7, 7, 7, 8, 9, 9, 9 has two modes, 7 and 9.

Measures of Spread characterize how spread out (or tightly packed together) the numbers are in a given data set.

- **Range** is the difference (literally, subtraction) between the highest and lowest numbers in a data set.
- **Standard Deviation** is a measure of variability (or consistency) within a set of data. The data set {4, 5, 6} would have a low standard deviation compared to the data set {1, 5, 9}, even though both sets have the same mean, median, and mode. You do not need to know how to calculate standard deviation.

(Note: this is a frequency distribution.
Know what it means and how it works!)

<i>Days of Rain per Month in Lone Desert, January 2003 to December 2006</i>	
Days of Rain	Number of Months
0	22
1	13
2	5
3	4
4	2
5	1
6	1

1. Determine the mean, median, mode, and range of the above data set.

Quiz Number	Jimmy's Score	Deanna's Score
1	84	95
2	82	73
3	83	82
4	85	77
5	80	95
6	83	80
7	84	79

2. Compare the mean, median, mode, range, and standard deviation of the following two data sets.

Statistics Post-Test

<i>Number of Moons of Solar System Planets and Dwarf Planets</i>	
Number of Moons	Number of Planets/ Dwarf Planets
0	4
1	2
2	2
5	1
13	1
27	1
62	1
63	1

1. Jupiter has 63 moons. If Jupiter were removed from the data set, would the mean, median, mode, or range be affected most?

2. The four gas giant planets have the four highest moon counts. What are the mean and median numbers of moons amongst the NON-gas giant planets?

Game Number	Gary	Michael
1	205	177
2	223	233
3	197	251
4	214	190
5	222	168
6	210	170
7	201	233
8	198	215

3. Compare the mean, median, mode, range, and standard deviation of the above two sets of bowling scores.

4. A prediction about which bowler's next score, made based on the information above, is more likely to be accurate? Why?

5. Based on the data provided, which bowler is more likely to bowl above 225? Above 200? Who is the better bowler?

Coordinate Geometry, Part 1

[MathOpenRef](#), [IXL \(Lines in the Coordinate Plane\)](#), [Kuta 1](#), [Kuta 2](#), [Kuta 3](#)

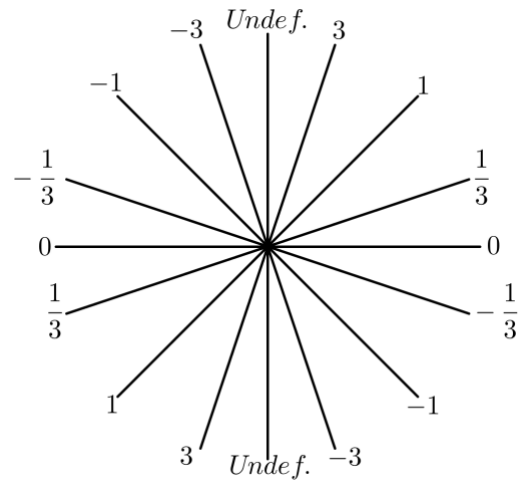
Slope measures the steepness of a line.

- *Positive* slopes go up and right.
- *Negative* slopes go down and right.
- *Zero* (0) slopes are horizontal.
- *Undefined* slopes are vertical.

The relationship between slope (m) and two points (x_1, y_1) and (x_2, y_2) is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

"Slope equals rise over run"



Some example slopes.

1. What is the slope between the points $(1, 4)$ and $(-6, -10)$?

2. If the slope between $(3, 2)$ and $(x, 14)$ is 4, what is x ?

Equations of a Line

Slope-intercept form: $y = mx + b$

To obtain: solve for y .

Slope = m y -intercept = b

Standard form: $ax + by = c$

To obtain: get x and y on same side.

Slope = $-\frac{a}{b}$ y -intercept = $\frac{c}{b}$

3. What is the slope of the line that fits the equation $3x - 5y = 15$? What is the y -intercept?

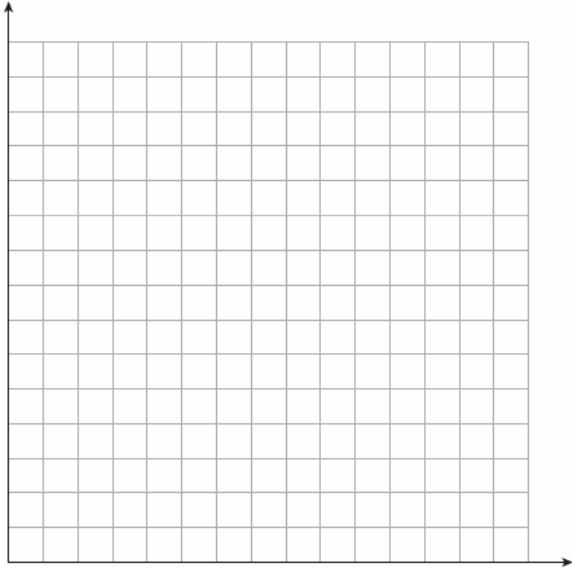
5. In standard form, write the equation of a line with slope $\frac{2}{3}$ and y -intercept -2 .

4. If $x - 2y = 4y + 7$, what is the slope and y -intercept of this equation's graph?

6. A line goes through the points $(-7, 5)$ and $(0, 2)$. What is its equation in standard form?

Coordinate Geometry Post-Test

1. Draw a depth (d) vs. time (t) graph for a swimming pool that is 8 feet deep, being drained by 1 foot every 10 minutes.



2. What is the slope, and what does it represent?

3. What is the y-intercept, and what does it represent?

4. What is the equation of the line?

5. Solve the equation to find the time at which the depth is 5'8".

6. Find the distance between the points $(-3, -5)$ and $(6, 7)$.

7. Find the slope of the line perpendicular to $3x - 2y = 7$

8. If John sells 5 cars this month, he will earn \$5,000. If he sells 11 cars, he will earn \$6,800. Assuming that the relationship between his sales and his earnings is linear, what is the equation for his earnings, E , in terms of the number of cars he sells, c ?

9. What is John's base monthly salary before he sells his first car?

10. How much does John earn for each car he sells?

Smart Math Strategy: Logic and Process of Elimination

You can use process of elimination to make educated guesses on many math questions. While your intuition is the most valuable tool, there are some guidelines you can follow to help you quickly eliminate answer choices on the math section:

- Answer choices on ACT math questions are often **very far apart**. In these cases, you can estimate and use process of elimination to easily get rid of several answer choices, and sometimes to answer the question without doing any actual calculations.
- The ACT does not claim to draw **diagrams to scale**, but it almost always does. Therefore, it is perfectly OK — and actually very helpful — to eyeball angles, relative lengths, and areas unless the diagram is clearly off-scale.
- Answers should always make **logical sense**. If a \$50 shirt goes on sale, its price will not be \$1.75, higher than \$50, or negative (not that you'll usually have those choices).

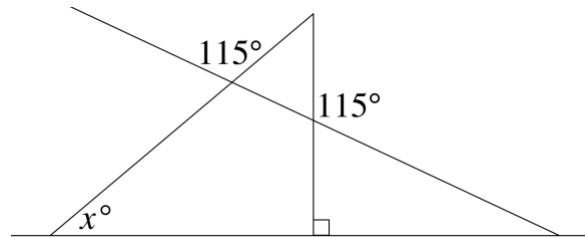
Directions: Eliminate clearly incorrect answers *without doing any actual work*.

1. If the price of a \$7,500 car is increased by 22.387%, what is the new price? Round your answer to the nearest dollar.

- (A) \$7,703
- (B) \$8,826
- (C) \$9,179
- (D) \$11,288
- (E) \$13,987

2. Franklin is twice as old as Zach. Obadiah is eight years older than Franklin. Ulysses is three years less than twice Obadiah's age. If Ulysses is 57, how old is Zach?

- (A) 1
- (B) 5
- (C) 11
- (D) 23
- (E) 54



3. What is x in the diagram above?

- (A) 40
- (B) 50
- (C) 60
- (D) 70
- (E) 80

4. The probability of a certain event is 30%. What is the probability it will occur three times in a row?

- | | |
|-----------|----------|
| (A) 0.27% | (B) 2.7% |
| (C) 9% | (D) 27% |
| (E) 90% | |

Reading: Answering Questions

Always start with Strategy #1 (unless it's a "Which of the following" question, of course).

- Remember that the information is always in the passage. You do not need outside knowledge or information to answer reading questions on the ACT. Even questions asking you to "infer" or say which answer is most "likely" still refer directly to information in the passage.
- However, even when you get line numbers, the information will NOT be exactly right there. It wouldn't be much of a test if a question referred to line 17 and the answer was right there.
- The information you need to answer the question is typically 2-4 lines before or after the lines given to you. For that reason, you should read the entire paragraph around the given lines, and if necessary also check out the end of the previous paragraph or beginning of the next one.

If Strategy #1 doesn't get you the answer, Strategy #2 is Process of Elimination.

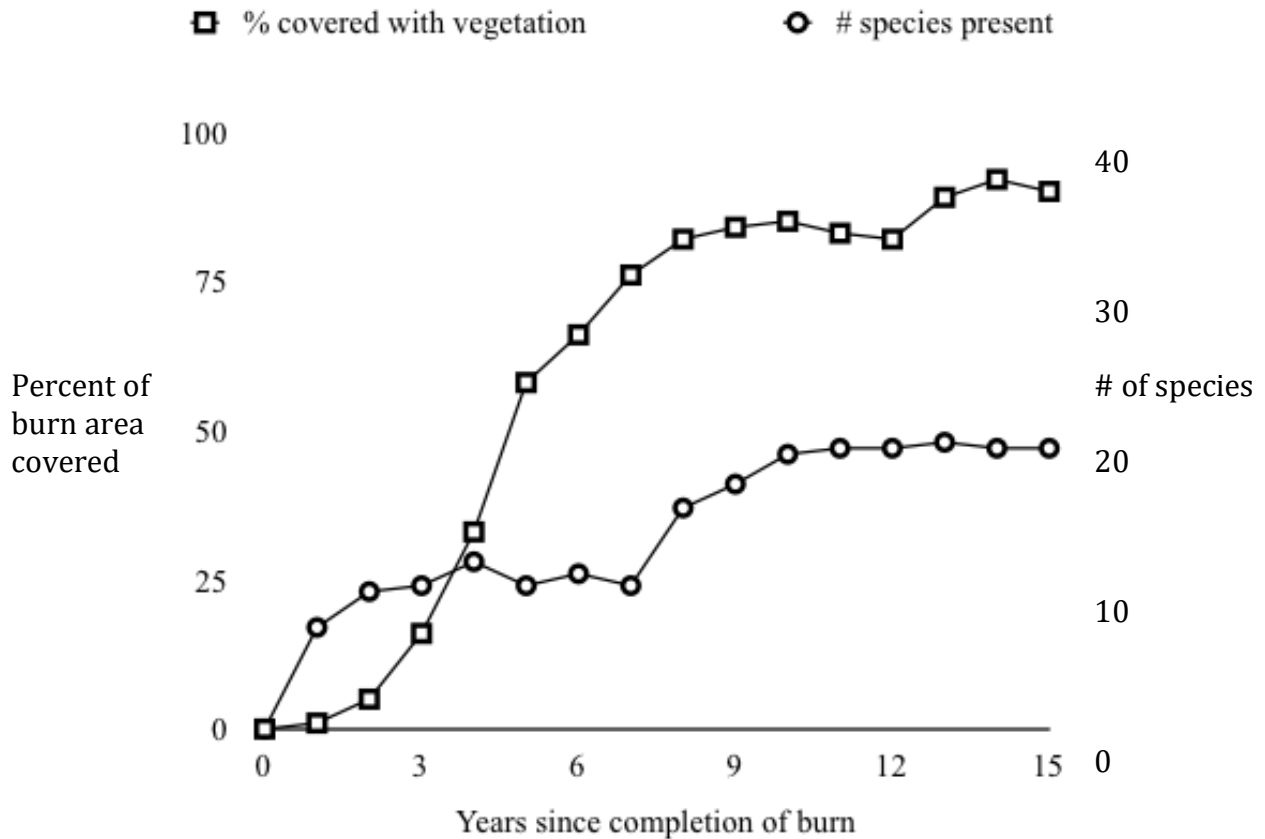
- You can almost always eliminate one or two answer choices that are clearly wrong.
- When you use Process of Elimination, you are looking for things that make the answer choices *wrong*, not things that make them right.
- When you're down to two answer choices:

 *The difference is one word that makes one of the answer choices wrong.* 

Answers that are part true and part false are always false. If an answer choice looks great but also makes you feel conflicted, it's wrong.

ACT Science: Reading ACT Figures and Tables

Many ACT graphs contain more than one data set, represented with different scales and labels. This example graph shows the percent of the area covered with vegetation and the number of species present at the site after an intentional burn.



1. How many species were present 10 years after the burn?

2. What is the significance of the point where the graphs cross?

3. When vegetation coverage first reached 60%, how many species were present?

4. If vegetation decreased, the number of species would most likely:

- A) increase, because more resources would be available.
- B) increase, because the number of species increases with plant coverage.
- C) decrease, because the number of species is poised to decrease anyway.
- D) decrease, because the number of species seems tied to the amount of vegetation coverage.

ACT: Approaching the Writing Test

The real Essay Task...

Essay Task

Write a unified, coherent essay about the prevalence of advertising in our society. In your essay, be sure to:

- clearly state your own perspective on the issue and analyze the relationship between your perspective and at least one other perspective
- develop and support your ideas with reasoning and examples
- organize your ideas clearly and logically [SEP]
- communicate your ideas effectively in standard written English

Your perspective may be in full agreement with any of those given, in partial agreement, or wholly different.

And the easiest structure:

1. Here's my point of view.
2. Here's why it's right.
3. Here are some examples.
4. Yes, there's this other stuff.
5. But in the end, I'm correct.

UNIT 3

Descriptive Clauses

[CCC 1](#), [CCC 2](#), [IXL 1](#), [IXL 2](#), [Chomp Chomp](#)

Descriptive Clauses are groups of words that describe a nearby noun. You might have heard other names for these such as adjective clauses, appositives, modifiers. Since you will not need to make these fine differentiations on the SAT, we will use the general term descriptive clauses. Some ways that you can often descriptive clauses are:

- Words ending in -ing without a verb in front. Example: “The birds flying slowly across the sky were both beautiful and powerful.”
- A parenthetical offset by commas/dashes. Example: “These birds, about as large as pigeons, looked so much more elegant than any bird I’d seen before.”
- That, than, which, who-what-where-when-why: “The birds that I’ve just described to you have remained in my mind since that day when I first them.”

The SAT will test you on one property of descriptive clauses: *they must describe the correct noun*. Descriptive clauses almost always describe the noun to which they are closest.

Determine whether the descriptive clause functions correctly in each sentence. If not, fix.

1. Happier than ever before, Julie’s dress was the perfect match for her radiant smile.
2. Gaston, never having been to Disneyland, realized upon growing older that an irreplaceable childhood experience had been denied to him.
3. Perfectly preserved on the moon’s surface, the astronauts who visited the moon left footprints that will likely be there for eons to come.
4. Having supported both candidates throughout their political careers, the election presented a dilemma for Freddie as they were matched against each other in the primary.
5. The Tigers’ quarterback, one of the best teams in the county, was badly injured.
6. Jesper looked ahead excitedly to Saturday’s Farmers’ Market, a day when he would have no work and few responsibilities.
7. The categorization of the tomato, generally considered to be a vegetable, is in fact technically a fruit.
8. Often one to procrastinate unnecessarily, the old barn sat for years before John finally got around to fixing it up.
9. Transliteration from Chinese to English, a confusing science to those who do not understand its myriad of rules, is nonetheless grounded in logic.
10. I found my car keys looking under the front seat of my car.

Subjects and Objects

[CCC 1](#), [CCC 2](#), [CCC 3](#), [IXL 1](#), [IXL 2](#), [Chomp Chomp](#)

Some grammar questions on the SAT test you on the difference between subjects and objects. For example, which of the following is correct?

“You don’t like Franklin and I, do you!” or “You don’t like Franklin and me, do you!”

If you don’t know the answer, you’ll learn it on this page. But first, definitions: A **subject** does something in a sentence. An **object** has something done to it. In English we use different words when something is a subject compared to when it is an object. Notice the differences between these two sentences about Jeremy, Lisa, and Jasmine:

“He saw them.” vs. “They saw him”

In the first sentence, “He” is a subject and “them” is an object. In the second sentence, “They” is a subject and “him” is an object. Same people, different words.

Subject	I	he/she	they	we	who	it	you
Object	me	him/her	them	us	whom	it	you

Tips about subjects and objects:

- Remove other people from the sentence. (Go back to the sentences at the top of this page: “You don’t like I!” or “You don’t like me” is an easy decision).

For “who” and “whom”...

- Is there a clear subject *after* the word? (Example: “Fred is the person to WHOM you need to speak,” or “WHOM shall I say is calling?”)
- The “he and him” trick: if you can replace the word, somewhat correctly, with “he,” then use who; if you can replace it with “him,” then use whom.
 - “To whom shall I address this letter?”
 - ➔ It makes more sense to say “to him” than “to he,” so we use “whom” instead of “who”.
 - “That’s the person who asked me the question.”
 - ➔ It makes more sense to say “he asked me” than “him asked me,” so we use “who” instead of “whom.”

Subjects and Objects Practice

Determine whether each underlined word is the correct subject/object choice; fix if necessary.

1. John is the one person towards who the manager was afraid to be as direct as she should have been.

6. Janelle didn't appreciate the mechanic being so gruff towards she and me.

2. Alejandra said that she would consider coming to the movies with Sophia and we as long as we didn't see anything scary.

7. Meanwhile, the mechanic thought it should be up to him whether to conduct additional tests or stick with his initial diagnosis of the car's problems.

3. After the end of Danica's game, her dad and she went to dinner to celebrate her success.

8. In the end, we realized that us customers are always right, and we took our business to another shop.

4. Despite the council's general disfunction, the decisions made by they and the mayor were generally quite wise.

9. Luckily, there was a repair shop right around the corner with a much nicer man to whom we were more than thrilled to take our poor little car.

5. We and our political allies were the big winners at the end of the election.

10. It was lucky that me and Janelle made that decision, because the second repair shop did a great job.

Subordinate Conjunctions and "Directional Words"

[Chomp Chomp](#)

sub-or-di-nate/sə'bôrdn-it/ Adj. Lower in rank or position: "his *subordinate* officers."

Subordination is the concept that sentences and paragraphs have ideas that must relate to each other in the correct way. "I want food because I am hungry" is correct; "I am hungry because I want food" is not correct, and is an example of incorrect subordination.

Proper subordination means correctly using **subordinate conjunctions**, such as:

- And
- Or
- Because
- Since
- Despite
- Although
- However
- Given that
- Along with
- In contrast with
- On the other hand
- As a result
- Nevertheless
- Still
- ...and many more

When you see any of these words, or other similar ones, ensure that they are connecting ideas in ways that makes sense in the context of the passage.

Determine whether each sentence demonstrates correct use of subordinate conjunctions:

1. Even though I had been to the museum three times before, I had seen most of the exhibits already.

4. Elizabeth seemed calm on the outside, yet on the inside she had to be very nervous.

2. Due to the lasting effects of my illness, I didn't feel my best on my vacation, although it was a full two weeks later.

5. The architect seemed unwilling to reconsider his plans for the building along with the fact that several engineers said it would be nearly impossible to actually build.

3. The machine made strange noises and rattled furiously, but something seemed wrong with it.

6. Given that Florida is such a warm place to live, Michigan can often be a very cold one.

However, Though, Although, But

[Explanation 1](#), [Explanation 2](#), [Exercise](#)

However:	adverb — "I like dogs. <u>However</u> , I don't like cats."
Though:	adverb — "I like dogs. I don't like cats, <u>though</u> ." conjunction — " <u>Though</u> I like dogs, I don't like cats."
Although:	conjunction — " <u>Although</u> I like dogs, I don't like cats."
But:	conjunction — "I like dogs, <u>but</u> I don't like cats."

Note that *though* and *although* are not always interchangeable. Also note that *however* cannot act as a conjunction to join two complete sentences.

Indicate **ALL** acceptable possibilities (out of those four words) for each blank. Answers below.

1. I like Diet Coke, _____ I don't like Coke Zero.
2. _____ nobody told me about the secret meeting, I found out about it anyway.
3. The hand sanitizer was placed very obviously on the table; nobody used it, _____.
4. My phone rang, _____ I didn't answer it.
5. I read through the pamphlet. _____, I still had questions.
6. I read through the pamphlet; I still had questions, _____.
7. He tried, _____ his explanation was not convincing.
8. _____ he tried, his explanation was not convincing.
9. Some animals have fur; whales, _____, do not.
10. The school claimed to have a beautiful campus; _____, its campus was very plain.
11. My jacket looked good _____ felt very uncomfortable.
12. There remained no doubt: _____ I wanted a dog, I would not be getting one.
13. I like most magazines. This one, _____, was kind of offensive.
14. I had no idea where she went, _____ I said I'd watch her dog until she got back.
15. _____ I've had three cups of coffee today, I'm suffering from only moderate jitteriness.

Run-Ons and Fragments

[CCC 1](#), [CCC 2](#), [IXL 1](#), [IXL 2](#), [IXL 3](#), [Chomp Chomp 1](#), [Chomp Chomp 2](#)

Run-ons and sentence fragments are defined by grammar, not length. A **run-on sentence** includes a complete sentence and another complete sentence or fragment which is not properly connected. While a **fragment** lacks a subject or a verb necessary to make it a complete sentence. (Get it? ☺)

Run-on or correct sentence?

___ 1. While I don't agree with everyone's take on the issue, and although I think that many people have uninformed opinions, I will admit that some people legitimately feel differently than I do despite what seems to me to be clear evidence supporting my side.

___ 2. Jake went to the mall, when he came home, it was dark.

___ 3. Lizzie told me that I could meet her at the mall, despite the fact that she had somewhere to go later, I got there late and she'd already left.

___ 4. Jonathan is a winner; his friend, for what it's worth, is a really nice guy.

A run-on sentence can be fixed by adding a:

- Period
- Semicolon
- Comma + conjunction (and, but, etc.)
- Or by completely rewriting the sentence

A comma alone creates a run-on; it does not fix one.

Fragment or correct sentence?

___ 5. It seems perfect.

___ 6. The answer to the question which is on the test.

___ 7. You would never want to go there if you had a choice.

___ 8. Whenever one attempts such a difficult procedure, especially as a new doctor, and despite modern technological advances in the procedure.

*How can *adding* words to a sentence make it a fragment? _____

Coordinate Geometry, Part 2

[MathIsFun Midpoint](#), [Distance](#), [Parallel/Perpendicular](#)

Midpoint

The midpoint between two points is simply the average of the two points (or specifically, the average of the x -coordinates and the average of the y -coordinates.)

1. What is the midpoint of the segment running from $(-1, -7)$ to $(3, 11)$?
2. If one endpoint of a segment is $(2, 13)$ and its midpoint is $(7, 15)$, what is the coordinate of the other endpoint?

Distance

The distance between two points on a graph can be calculated using the distance formula:

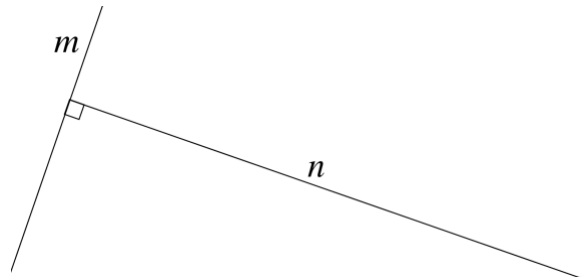
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. This formula is equivalent to the Pythagorean theorem, so *treat distance questions as right triangle questions* to get the answer more quickly!

3. What is the distance between the points $(3, 1)$ and $(-3, 9)$?
4. If x is a negative integer and the distance between $(x, 5)$ and $(2, 17)$ is 13, what is x ?
5. Amos' office is 4 miles east of his house. During the course of the workday he leaves his office and drives 3 miles south, 6 miles east, another 3 miles south, and finally 2 miles west. After this driving, how far is Amos from his house?
6. What is the distance between $(-7, 13)$ and $(-14, -11)$?

Parallel and Perpendicular Lines

Parallel lines' slopes are *equal*. Perpendicular lines have *opposite reciprocal* $\left(-\frac{1}{x}\right)$ slopes.

7. What is the slope of a line (a) parallel and (b) perpendicular to $y = 4x - 7$?
8. What is the slope of a line (a) parallel (b) perpendicular to $3x + 9y = 13$?
9. What is the value of m if the slope between $(m, 5)$ and $(8, m)$ is $-\frac{2}{5}$?



10. If neither line m nor line n is parallel to the y -axis, what is the product of the slopes of lines m and n ?

Percentages and Percent Change

[IXL 1](#), [IXL 2](#), [Read the directions](#), [Kuta](#), [MathIsFun](#)

$$\frac{\%}{100} = \frac{\text{new, after, "is", finishing value}}{\text{old, before, "of", starting/comparison value}}$$

This is how percentages work: start at one number, then go up or down to another. This setup works for ALL percent questions because the definition of percent is $\frac{\quad}{100}$.

- Elizabeth made \$60,000 last year and paid 20% in taxes. How much money did she pay in taxes?
- The dosage of a medication is reduced from 750 mg to 600 mg. What percent of the old dose is the new dose?
- Fred and Kris were on a hike. Fred drank 30 ounces of water, which was 37.5% of the total amount of water Kris consumed. How much did Kris consume?
- Joe paid \$20,160 dollars in taxes last year. If he made \$72,000, what was his overall tax rate?
- Which of the following represents 4% of 0.2% of x ?
(A) $0.000008x$ (B) $0.00008x$
(C) $0.0008x$ (D) $0.008x$ (E) $0.08x$
- Liza paid \$42 for electricity this month, which was 70% of what she paid last month. How much was last month's bill?
- By what should a quantity be multiplied so that the result is 8.3% of the original value?

How should you solve percent change questions?

Most of the percent questions you will see on the SAT are *percent change* questions. In a percent change question, *everything starts at 100%*!

A town's population went from 35,000 to 56,000. By what percent did the population increase?

$$\frac{x}{100} = \frac{56,000}{35,000}$$

$x = 160$. Since every percent change starts at 100, the increase is 60%.

Percent Change Practice

8. A pair of socks is \$14 after a 30% discount. What was its original price?

11. By what percent must Fred increase the number of stamps in his collection if he currently has 76 stamps and wishes to have 209?

9. The price of a \$40 sweater is increased by 20%. What is the new price?

12. If the population of a town increases by 10% one year and then decreases by 10% the next year, what is the overall percent change for the two years?

10. The value of an investment decreases from \$17,200 to \$15,738. By what percentage is the value of the investment decreased?

13. The base of a triangle is stretched by 50% and the height of a triangle is raised by 20%. What is the percent increase in the area of the triangle?

14. Counting the 8% sales tax, the total price of a computer was \$1,566. What was the price before tax?

17. Townsville received 20% more rain in 1998 than 1997, and 30% less rain in 1999 than 1998. If Townsville received 50.4 inches of rain in 1999, how much rain did it receive in 1997?

15. The price of a \$900 refrigerator is to be reduced to \$765. By what percent must the price be reduced?

18. If the number of bacteria in a colony jumps from 150 to 1,125, by what percent has the number of bacteria increased?

16. Fred got a 10% raise in 2009. For 2009 and 2010, his salary went up by 26.5% overall. How much of a raise did Fred get in 2010?

19. A certain quantity is multiplied by $\frac{5}{6}$ and then divided by $\frac{2}{3}$. By what percent must this result be decreased to return the quantity to its original starting value?

Percentages Post-Test

1. In a certain month, a store made a profit of \$6,972, which was 42% of their gross revenue. What was the store's gross revenue?

3. A survey in January indicated that the residents of Temporia supported Candidate A over Candidate B by a 3-to-1 ratio. In February, Candidate B increased his number of supporters by 30%. If Temporia has 15,000 adults, how many of them were supporters of Candidate B at the end of February?

2. Following a 15% discount, the price of a new washing machine is \$629. What was the original price?

4. The number of homes subscribing to cable TV in a certain county has decreased by 10% each year for the past three years. What is the percent decrease in cable TV subscribers in this county over the past three years?

Unit Conversion, A.K.A. Dimensional Analysis

[MathIsFun, Online](#)

The SAT will ask you to convert measurements from one set of units to another. You will do this by using dimensional analysis, which utilizes multiplying to cancel and replace units.

Write everything with a numerator and denominator. First fill in the units, canceling and replacing as necessary to achieve the proper units. The numerator and denominator of each individual fraction must represent an equivalency!

Ex. A: Convert 18 inches per second to feet per minute:

$$\left(\frac{18 \text{ inches}}{1 \text{ second}}\right) \times \left(\frac{\text{feet}}{\text{inches}}\right)$$

$$\left(\frac{18 \text{ inches}}{1 \text{ second}}\right) \times \left(\frac{\text{feet}}{\text{inches}}\right) \times \left(\frac{\text{seconds}}{\text{minute}}\right) = \frac{\text{feet}}{\text{minute}}$$

$$\left(\frac{18 \text{ inches}}{1 \text{ second}}\right) \times \left(\frac{1 \text{ foot}}{12 \text{ inches}}\right) \times \left(\frac{60 \text{ seconds}}{1 \text{ minute}}\right) = \frac{90 \text{ feet}}{1 \text{ minute}}$$

1. Convert 74 cents per foot to dollars per yard.

4. Convert 15 square feet per person into square yards per dozen people.

2. Convert 11 grams of fat per quarter-pound serving into calories from fat per pound. A serving is 1/4 of a pound. (Note: 1 gram of fat = 9 calories.)

5. Convert 8 grams per cubic inch to kilograms per cubic foot. (Note: 1 kilogram = 1,000 grams.)

3. Convert 25 minutes per day into days per year.

6. In the ancient Greek system of measuring lengths, 1 *dichas* was equivalent to 8 *daktyloi* and also to $\frac{1}{3}$ of a *pechys*. If 1 *pechys* is approximately 18.2 inches, approximately how many inches are in 30 *daktyloi*?

Rates and Dimensional Analysis Post-Test

1. Three years ago, the Lions' Facebook page had 13,325 fans. Now, it has 23,075. The Rockets had 7,550 fans three years ago, but their fan base is growing at a rate 50% faster than that of the Lions. If both fan bases experience linear growth, How many fans do the Rockets have now?

2. Halley's Comet is moving at approximately 158,000 miles per hour when it passes earth. Convert this speed to miles per second.

3. Convert 44 miles per hour to feet per second. (Note: 1 mile = 5,280 feet.)

4. The density of an object is 24 pounds per cubic foot. Convert this measurement into ounces per cubic inch. (Note: 1 pound = 16 ounces.)

Two-Way Tables

[IXL 1](#), [IXL 2](#), [Khan Academy](#)

Two-way tables, like the ones below, can be used to evaluate the presence of relationships between two variables and to evaluate conditional probability.

Table A

	Pet Preference		Total
	Dogs	Cats	
Boys	27	13	40
Girls	41	19	60
Total	68	32	100

1. What is the probability that a given boy prefers dogs to cats? That a given girl prefers dogs to cats?

2. Does the table provide evidence for a relationship between sex and pet preference?

Table B

		Eye Color		Total
		Light	Dark	
Hair Color	Light	17	25	42
	Dark	19	46	65
	Total	36	71	107

3. Given that a student has light hair, what is the probability that he or she has light eyes?

4. Given that a student has dark eyes, what is the probability that he or she has dark hair?

5. Does the table provide evidence for a relationship between hair color and eye color?

Table C

		Fav. Food		Total
		Junk	Upscale	
Fav. Drink	Soda	46	22	68
	Water	7	7	14
	Other	13	65	78
	Total	66	94	160

6. Given that an individual's favorite food is upscale food, what is the probability that his or her favorite drink is soda?

7. Does the table provide evidence that individuals who prefer junk and upscale food are equally likely to select water as their favorite drink?

8. Of individuals whose favorite drink is soda or water, what percentage said that they preferred upscale food?

9. Does the table provide evidence for a relationship between favorite food type and preference for soda compared to non-soda drinks?

10. Based on the table, in a group of 16 people who preferred upscale food, how many of them would prefer a drink other than soda or water?

Solving Linear Equations/Inequalities in Terms of Other Variables

[Worksheet, PurpleMath, MathIsFun](#)

Isolate the variable you're interested in on one side of the equation. Whatever you have on the other side is the answer.

Ex. A:

$$y = mx + b; \text{ solve for } m$$

$$y = mx + b$$

Subtract b :

$$y - b = mx$$

Divide by x :

$$\frac{y - b}{x} = m$$

Ex. B:

$$a^2 + b^2 = c^2; \text{ solve for } b$$

$$a^2 + b^2 = c^2$$

Subtract a^2 :

$$b^2 = c^2 - a^2$$

Root both sides:

$$b = \sqrt{c^2 - a^2}$$

1. $F = \frac{9}{5}C + 32; \text{ solve for } C$

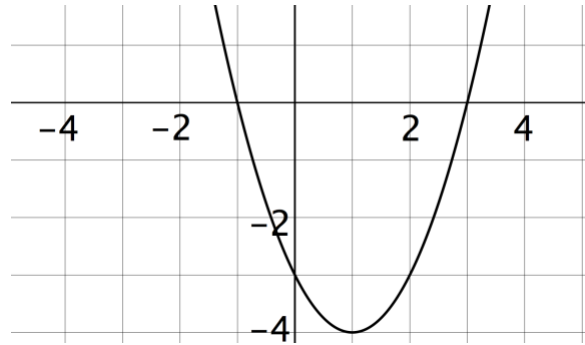
3. $a = \frac{v_f - v_i}{t}; \text{ solve for } v_i$

2. $V = \frac{4}{3}\pi r^3; \text{ solve for } r$

4. $r = \frac{4x-3}{x}; \text{ solve for } x$

Relating Functions and Their Graphs

The goal of this page is for you to understand the relationship between functions, factors, roots, solutions, zeroes, x -intercepts and graphs. Consider the graph, shown at right, of $y = x^2 - 2x - 3$. What are some of the ways that we can think about the relationship this graph at its equation?



Graphs consist of all the points that make an equation true.

Notice, for example, that $(2, -3)$ is a point on the graph. That's because when we plug in that point, we get $-3 = 2^2 - 2(2) - 3$; this simplifies to $-3 = -3$. The point $(2, -3)$ gets to be on the graph *because it makes the equation true*.

Graphs are input(x)/output(y) visualizations.

If we plug in any x -value, we can find the corresponding y -value, or the value of the function $f(x) = x^2 - 2x - 3$. The graph shows all of these corresponding values.

The x -intercepts are zeroes, roots, and solutions.

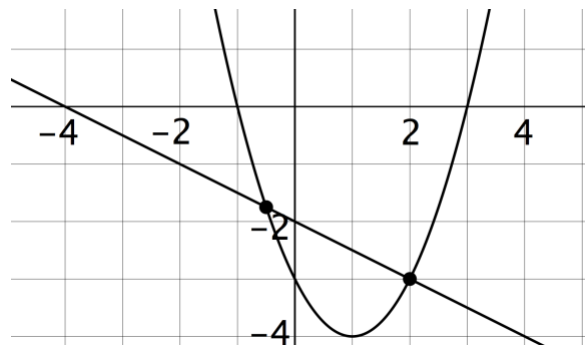
Notice that $x = -1$ and $x = 3$ both result in y values of 0. This means that -1 and 3 are both *zeroes* or *roots* of the function: they are x -values that make the value of the function exactly zero. If this we solved $x^2 - 2x - 3 = 0$, then -1 and 3 would both be *solutions*.

We can write functions to fit a graph.

Imagine we didn't know the original equation and just looked at this graph. Since this graph has two zeroes, it also has two factors. *Each zero of the graph gives us a factor that equals zero*. Therefore $x = -1$ gives us $x + 1 = 0$, and the $x = 3$ gives us $x - 3 = 0$. Now we know that the two factors are $(x + 1)$ and $(x - 3)$. Multiplying them gives us our original polynomial, $x^2 - 2x - 3$.

Intersections

Now let's add one more element: the equation $y = -\frac{1}{2}x - 2$ and its graph. The notes above all still apply (though we don't generally talk about roots/zeroes of linear equations). But now we have *intersections*. Intersections lie on both graphs; they make *both equations true*. An intersection is the same thing as a solution to a system of equations: a set of x - and y -values that makes both equations true. To find where two graphs intersect, set the expressions equal to each other and solve!



More About 2-by-2 Systems of Linear Equations

[PurpleMath](#), [MathIsFun](#), [Algebra.com](#), [One More](#)

There are three types of systems of equations:

Type	Solutions	Equations			Graphs
		If you solve...	To create them...	Example	
Independent	1	You get x - and y -values	$\frac{a_1}{b_1} \neq \frac{a_2}{b_2}$	$x + 3y = 7$ $x + 5y = 6$	Different slopes; intersect at a point
Dependent	Infinite	You get an identity, like $3 = 3$	$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{c_1}{c_2}$	$x - 3y = 2$ $3x - 9y = 6$	The same line; they exactly overlap
Inconsistent	0	You get a falsehood, like $0 = 2$	$\frac{a_1}{b_1} = \frac{a_2}{b_2} \neq \frac{c_1}{c_2}$	$x + 2y = 7$ $2x - 4y = 1$	Two separate parallel lines

$$\begin{aligned} 3x - jy &= 7 \\ kx - 4y &= 14 \end{aligned}$$

Ex. A: If the system of equations shown above has an infinite number of solutions, what is the value of $j + k$?

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{c_1}{c_2} \qquad \frac{3}{k} = \frac{-j}{-4} = \frac{7}{14}$$

Cross-multiply in pairs to get $j = 2$, $k = 6$, and $j + k = 8$

$$\begin{aligned} 3x + 4.5y &= c \\ bx + 3y &= -7 \end{aligned}$$

1. If the system of equations shown above has an infinite number of solutions, what is the value of $b + c$?

$$\begin{aligned} 4.2x - 6y &= 3.9 \\ 7x - my &= c \end{aligned}$$

2. If the system of equations shown above has no solution, what CANNOT be the value of c ?

$$\begin{aligned} ax - y &= c \\ 2y - 7x &= b \end{aligned}$$

Ex. B: If the system of equations shown above has no solution, what is a ?

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} \qquad \frac{a}{2} = \frac{-1}{-7}$$

Cross-multiply to get $a = \frac{2}{7}$

$$\begin{aligned} ax + by &= 7 \\ 4x - 3y &= c \end{aligned}$$

3. If the system of equations shown above has no solution, what is the ratio of a to b ?

$$\begin{aligned} ax - 8y &= 4 \\ by - 6.25x &= 5 \end{aligned}$$

4. If the system of equations shown above has no solution, is the value of b ?

More Statistics

How do we know if data is reliable?

When we get data from a survey, experiment, or other type of study, we evaluate it in terms of its *reliability*: whether the data is very likely to accurately represent reality, and whether we would be smart to base important decisions on it. There are three main characteristics that make a data set *reliable*:

1. **Size:** the larger a data set, the less likely it is that any observed results are due to chance. It's easy for 2 random things to happen, but hard for 500 random things to happen. *Note: population size does not significantly affect needed sample size!*
2. **Representativeness:** We can only *generalize* the results of a study if the participants (or other data points) *accurately represent the larger group we're interested in*; a survey taken at the Marina Whole Foods would not accurately represent the views of all San Franciscans. To ensure *representativeness*, we almost always need *random sampling*. Random sampling means that *every member of the group we're talking about has an equal chance of being selected* into the survey or experiment.
3. **Consistency:** A small standard deviation and a tight pattern within the data make for more reliable data. If Julie gets test scores of 55%, 75%, and 95%, we're confused about her abilities. If she gets scores of 76%, 73%, and 73% — giving her the exact same average — we feel much more certain that she is in fact a C student.

What is a confidence interval?

A confidence interval applies when we are taking a *sample* of a population and don't know the *actual* population measurement, or *parameter*. A confidence interval, therefore, is similar to a *margin of error*; it provides a range within which we are some-percent certain (usually 90%, 95%, or 99% certain) that the true result actually lies. You will not need to calculate confidence intervals, but you should know what they mean.

For example: assume we take a survey of 1,000 likely voters and find that 63% of them support Proposition 2. Because we had so many respondents, we can be pretty sure our 63% result is a good one. In fact (using a complex formula), there is a 95% chance that the actual percentage of likely voters who support Proposition 2 is $63 \pm 3\%$. In other words, our 95% confidence interval is between 60% and 66%; we are 95% certain that the actual percent of likely voters who support Proposition 2 is between 60% and 66%.

On the other hand, if our survey only included 100 voters, our result is less reliable. The actual statistic here is that our 95% confidence interval is between 53% and 73% — in other words, there's a 1-in-20 chance that our survey was wrong by more than 10%! A smaller confidence interval is always better.

Confidence intervals are determined by the three factors listed above: the size of the sample; the representativeness of the sample; and the consistency of the data.

Completing the Square

[PurpleMath](#), [MathIsFun](#), [Kuta 1](#), [Kuta 2](#)

This process might come up on more difficult questions involving quadratic equations, equations of circles, and vertex form of equations of parabolas.

Ex. A: Rewrite the function $f(k) = x^2 - 6x + 20$ in a format that clearly indicates the minimum value of the function.

Group the x -terms together:

$$f(k) = (x^2 - 6x \quad) + 20$$

Add $\left(\frac{1}{2}b\right)^2$ inside the parentheses to complete the square, and subtract it outside the parentheses to maintain equality with the left side:

$$f(k) = (x^2 - 6x + 9) + 20 - 9$$

Simplify:

$$f(k) = (x - 3)^2 + 11$$

1. Rewrite the function $g(x) = x^2 + 12x + 31$ in a format that clearly indicates the minimum value of the function.

Ex. B: Rewrite the equation of the circle $x^2 + y^2 - 8x + 2y + 8 = 0$ in a format that clearly indicates the center and radius.

Group x - and y -terms together:

$$(x^2 - 8x \quad) + (y^2 + 2y \quad) + 8 = 0$$

Add $\left(\frac{1}{2}b\right)^2$ inside each set of parentheses to complete the square, and subtract it outside the parentheses to maintain equality with the right side:

$$(x^2 - 8x + 16) + (y^2 + 2y + 1) + 8 - 16 - 1 = 0$$

Simplify and move constant term:

$$(x - 4)^2 + (y + 1)^2 = 9$$

2. Rewrite the equation of the circle $x^2 + y^2 + 10x - 4y = -4$ in a format that clearly indicates the center and radius.

Factoring by Grouping[PurpleMath](#), [Examples](#), [Kuta](#)

Factoring by grouping can be used to factor polynomials with four terms.

Ex. A: Find all of the zeroes of the function

$$f(x) = x^3 + 3x^2 - 4x - 12.$$

Group the terms whose degrees are different by one:

$$f(x) = (x^3 + 3x^2) + (-4x - 12)$$

Factor the GCF from each group:

$$f(x) = x^2(x + 3) + (-4)(x + 3)$$

Add, grouping the coefficients:

$$f(x) = (x^2 - 4)(x + 3)$$

Continue factoring:

$$f(x) = (x + 2)(x - 2)(x + 3)$$

Solve:

$$x = -2, 2, \text{ or } -3,$$

1. Find all of the zeroes of the function

$$f(x) = x^3 + 4x^2 - x - 4.$$

Ex. B: Find all of the zeroes of the function

$$f(x) = 2x^3 - 3x^2 - 18x + 27.$$

Group the terms whose degrees are different by one:

$$f(x) = (2x^3 - 3x^2) + (-18x + 27)$$

Factor the GCF from each group:

$$f(x) = x^2(2x - 3) + (-9)(2x - 3)$$

Add, grouping the coefficients:

$$f(x) = (x^2 - 9)(2x - 3)$$

Continue factoring:

$$f(x) = (x + 3)(x - 3)(2x - 3)$$

Solve:

$$x = -3, 3, \text{ or } \frac{3}{2}$$

2. Find all of the zeroes of the function

$$g(x) = 3x^3 - x^2 - 75x + 25.$$

Right Triangles

The Pythagorean Theorem: $a^2 + b^2 = c^2$

[MathIsFun](#), [PurpleMath](#), [Kuta](#)

1. The two legs of a right triangle have lengths 5 and 12. How long is the hypotenuse?

3. Two sides of a right triangle are 7 units and $\sqrt{65}$ units long. What are all possible lengths of the third side of the triangle?

2. The longest side of a right triangle is 50 cm long. If one leg is 14 cm, how long is the other leg?

4. A right triangle has a height of 8 in and its area is 24 in^2 . What is its perimeter?

Pythagorean Triples

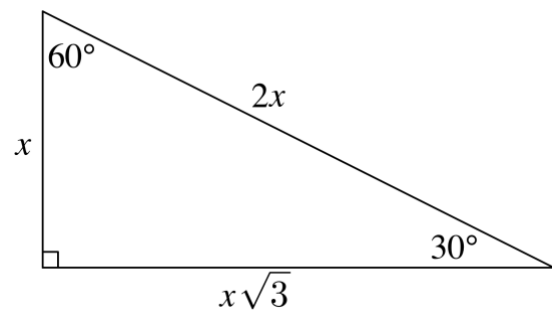
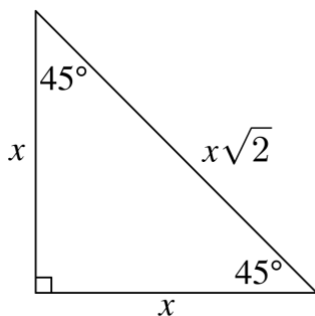
[MathIsFun](#), [Worksheet](#)

5. How far apart are opposite corners of a 9" by 12" piece of paper?

6. An airplane flies 250 miles, finishing north and east of where it started. If it finishes 70 miles east of where it started, how far north is it from its starting point?

Special Right Triangles

[Kuta 1](#), [Kuta 2](#), [IXL 1](#), [IXL 2](#), [IXL 3](#), [Online Help](#), [More Help](#)

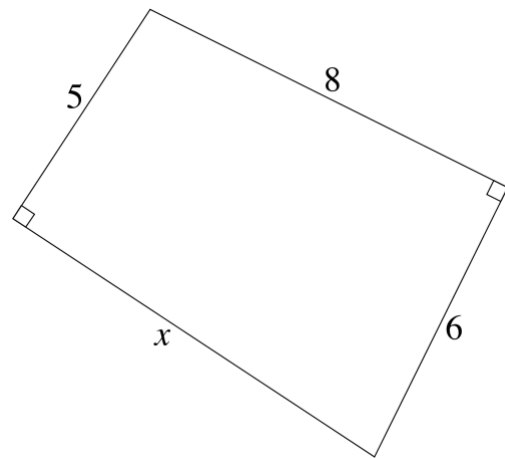


Right Triangle Practice

7. If one leg of a 45-45-90 triangle is 5 mm, how long is the hypotenuse?

12. The hypotenuse of a 30-60-90 triangle is 8 units. What is its area?

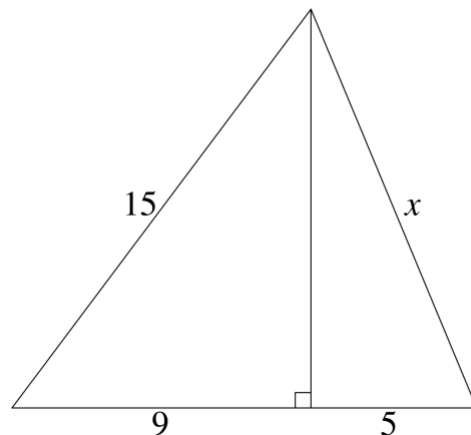
8. If the hypotenuse of an isosceles right triangle is 6 inches, how long is each leg?



9. If the hypotenuse of a 30-60-90 triangle is 12 feet, how long is each leg?

13. In the diagram above, what is x ?

10. If the leg of a right triangle opposite a 60° angle is 6 cm, how long are the other sides?



11. The area of a 30-60-90 triangle is $18\sqrt{3}$. What is the length of its hypotenuse?

14. In the diagram above, what is x ?

Similar Triangles

[MathOpenRef](#), [Kuta 1](#) [Kuta 2](#), [MathIsFun](#), [IXL](#)

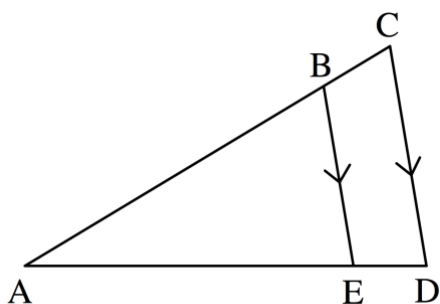
Triangles are similar if their angles are *the same* and their sides are *proportional*. In other words, similar triangles are the same shape, but different sizes.

How can you prove that triangles are similar?

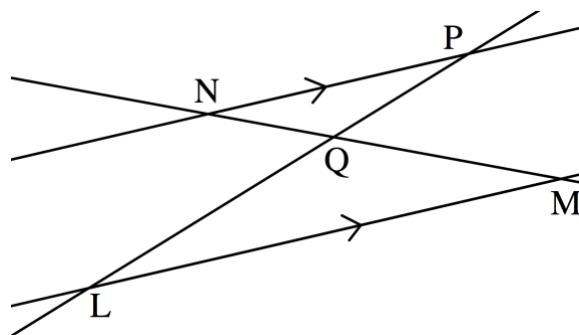
There are three ways:

1. At least two of the angles are the same (AA).
2. Two side-pairs are in equal proportion, and the middle angles are the same (SAS).
3. All three side-pairs are in equal proportion (SSS).

Two particular arrangements always lead to similar triangles; watch for them. *Anytime you see multiple triangles and parallel lines, look out for similar triangles!*



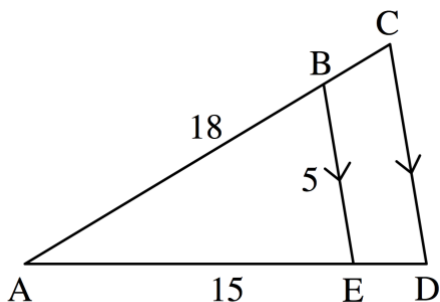
First is a line inside of a triangle, parallel to one of the sides. Triangle ABE is similar to triangle ACD.



Second is triangles sharing a common vertex, formed between parallel lines. Triangle LQM is similar to triangle PQN.

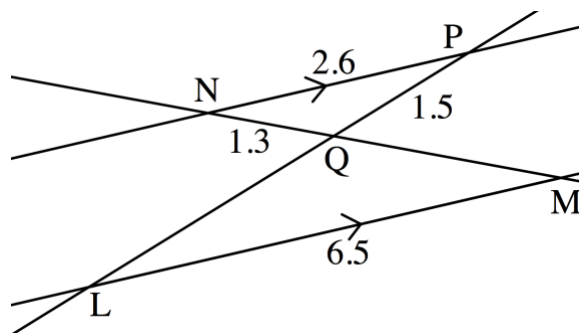
How do you solve questions involving proportional sides?

Anything with the word "proportion" is solved by using ratios and the setup $\frac{a}{b} = \frac{c}{d}$.



Ex. A: In the figure above, how long is CD ?

$$\frac{5}{15} = \frac{CD}{18}, \text{ then solve for } CD = 6$$



Ex. B: In the figure above, how long is LQ ?

$$\frac{2.6}{6.5} = \frac{1.5}{LQ}, \text{ then solve for } LQ = 4.75$$

Congruent Triangles

[Kuta](#), [MathIsFun 1](#), [MathIsFun 2](#), [IXL 1](#), [IXL 2](#), [IXL 3](#)

Triangles are *congruent* when they are exactly the same. Their corresponding angles are the same measure and their corresponding sides are the same length.

How can you prove that triangles are similar?

There are four ways:

1. All three side-pairs are the same (SSS).
2. Two equal side-pairs sandwich equal angles (SAS).
3. Two equal angles sandwich an equal side-pair (ASA).
4. Two equal angles are followed by an equal side-pair (AAS).

There is also a special rule for *right triangles*: if two corresponding side-pairs are equal, regardless of which ones, the right triangles are congruent. You might know this as HL (hypotenuse-leg), but LL (leg-leg) works too!

Once you find that triangles are congruent, most of your work is done. Just make sure you're matching up the proper sides — especially if one of the triangles is flipped, inverted, or rotated from the other.

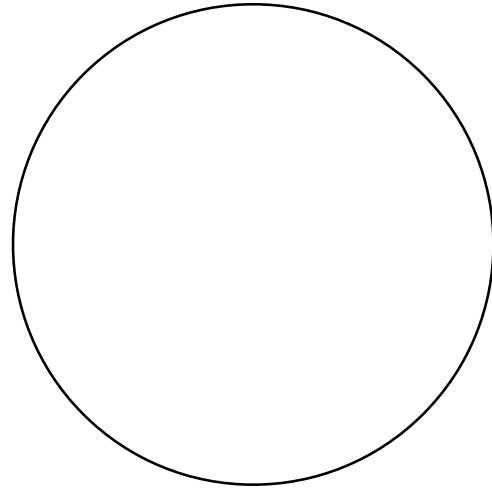
Circles

Circles are easy: they're all the same!

If you know any of

Radius, Diameter, Circumference, Area

...you know them all.



Formulas:

$$D = 2r$$

$$C = \pi d \text{ or } c = 2\pi r$$

$$A = \pi r^2$$

1. The radius of a circle is 6. Find its:

- Diameter
- Circumference
- Area

3. A circle's circumference is 8π . Find its:

- Radius
- Diameter
- Area

2. A circle's diameter is $4\sqrt{3}$. Find its:

- Radius
- Circumference
- Area

4. The area of a circle is 48π . Find its:

- Radius
- Diameter
- Circumference

Exponents and Roots

(Kuta 1, Kuta 2, Kuta 3, MathIsFun, IXL 1, IXL 2, many more!)

If you want to achieve a high score, know your exponent rules and be able to apply them.

	<u>Your Guess</u>	<u>Actual Rule</u>
Level 1:		
$x^a x^b$	x^{a+b}
$(x^a)^b$	x^{ab}
$\frac{x^a}{x^b}$	x^{a-b}
x^0	1
x^1	x
$x^{\frac{1}{n}}$	$\sqrt[n]{x}$
Level 2		
$\frac{x^a}{x}$	x^{a-1}
$x^{\frac{a}{b}}$	$\sqrt[b]{x^a}$
$x(x^a)$	x^{a+1}
x^{-a}	$\frac{1}{x^a}$
$a^x b^x$	$(ab)^x$
$\frac{1}{x^{-a}}$	x^a

Linear Equations and Inequalities Post-Test

1. $4.5x + 3(x - 1) = 12$

6. Given the formula for the surface area of a rectangular prism, $A = 2lw + 2hw + 2lh$, solve for h in terms of l and w .

2. $7(x + 4) - (x + 1) = -3(-2x - 9)$

7. Three sodas and two hot dogs cost \$15.50. Four hot dogs and five sodas cost \$28.50. How much are hot dogs and sodas individually?

3. $3(7 - x) + 2 < 8$

4. The cost in dollars, C , of a pizza with a certain number of toppings is $C = 9 + 1.25(x - 1)$. Select all of the true statements:

- I. A pizza with no toppings is \$9.00
- II. Toppings are \$1.25 each.
- III. Unlimited toppings are \$9.00.

8. A coffee shop pays \$600 each day in salaries, rent, and other fixed costs. Each cup of coffee they make costs them \$0.50 and sells for \$2.00. Each day, they give away three cups of coffee for free. Write an equation to determine the number of cups of coffee, c , the shop must *sell* to break even; then solve the equation.

5. Write and solve an equation for the number of toppings a person with \$14 could get on a single pizza.

Functions

[PurpleMath](#), [MathIsFun](#), [Kuta 1](#), [Kuta 2](#)

Here are two ways to think of functions. Choose whichever one works best for you. You must understand how functions work, so use as many resources as necessary until you do! Fundamental to your understanding is that $f(x)$, $g(x)$, $r(x)$, etc. all mean y !

A. Functions are *patterns* or *processes* that you're asked to follow. You're usually given a pattern with x , and you need to follow this pattern with other things in place of x . For example:

$$f(x) = 5x + 11$$

...is a process; when you're given a number, you multiply it by 5 and then add 11.

Compare $f(x)$ to $f(2)$. The expression $f(2)$ means you follow the same process but now with 2 instead of x . Multiply 2 by 5, add 11, and you get 21.

$f(9)$ means you multiply 9 by 5, add 11, and you get 56. If you're asked for $f(\text{anything})$, you multiply (anything) by 5 and add 11.

If $f(x) = 26$, it means that some number x , multiplied by 5, plus 11, gives you 26.

B. Functions are *places to plug in numbers and letters*.

Consider the same example as above for $f(x)$:

$$f(x) = 5x + 11$$

If you're asked for $f(2)$ or if $x = 2$, physically replace x with **2**:

$$f(2) = 5 \cdot 2 + 11$$

If you're asked for $f(x - 3)$, physically replace x with $(x - 3)$:

$$f(x - 3) = 5(x - 3) + 11$$

Look one more time at the original function:

$$f(x) = 5x + 11$$

If you're told that $f(x) = 26$, physically replace $f(x)$ with **26**:

$$26 = 5x + 11 \text{ (solve, } x = 3)$$

1. If $f(x) = \frac{3}{8}x - 10$, what is $f(16)$?

3. Given that $f(x) = (x + 7)^2$, what is x if $f(x) = 16$?

2. If $g(x) = x^2 - 3x$, what is $g(2x)$?

4. If $g(x) = x^2 + k$ and $g(-3) = 12$, what is k ?

Functions Practice

5. If $f(x) = 5x^2 - x + 4$, what is the value of the function when $x = 3$?
6. If $f(x) = x^2 - 3x + 4$, for what values of x does $f(x) = 8$?
7. If $f(x) = x^2 + h$, and $f(3) = 11$, what is the value of h ?
8. If $f(x) = x^2$, which of the following is equal to 12?
 (A) $f(\sqrt{12})$
 (B) $f(6)$
 (C) $f(12)$
 (D) $f(144)$
9. If $f(x) = x^2$ and $g(x) = 2x + 1$:
 a. What is $f(g(x))$?
 b. What is $g(f(x))$?
10. What is $x^3 + x + (1/x)$ when $x = 3$?
11. If $f(x) = x^2 - 13$, what is the least value of x for which $f(x) = 12$?
12. If $f(x) = x^2 - 11x + 24$, what is one value of x for which $f(x) < 0$?
13. If $f(x) = 2x^3$ and $f(-3x) = -2$, what is x ?
14. Given $g(x) = mx^2 - 4$, and $g(11) = 359$, what is m ?
15. If $r(x) = \frac{4x}{4-x}$, what is $r\left(\frac{1}{x}\right)$?

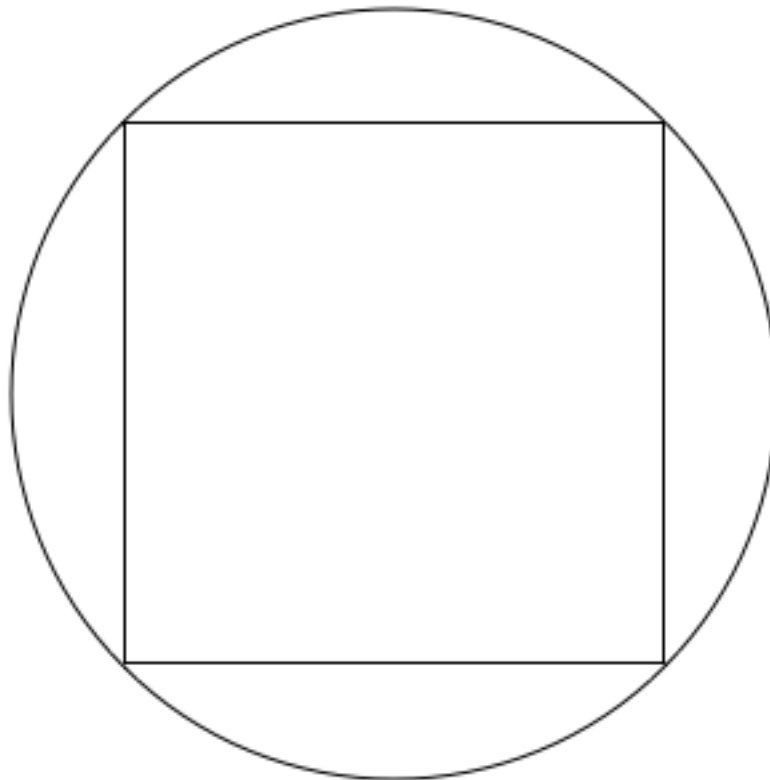
Smart Math Strategy: Three Steps for Geometry Questions

DRAW the situation - large, clearly, and accurately.

LABEL everything you can - angles, distances, relationships.

Write out **FORMULAS** for everything you have, everything you need, and anything else that might be relevant.

- Focus on “bells and sirens” such as 30° , 45° , 60° , and 90° angles, right triangles, anything with a $\sqrt{2}$ or $\sqrt{3}$, Pythagorean Triples, and parallel lines.
- A good tip (but also something to be careful about) is that when all of the shapes in a diagram are normal, symmetrical, parallel, etc., you can more safely make assumptions about elements of the diagram.



A square is inscribed in a circle. The area of the square is 36. What is the area of the circle?

Probability Distributions

The left column of a probability distribution is the data point that was measured/observed. The right column of a probability distribution is the percentage/probability/proportion of the time that the specific data point happened. Don't worry about how the columns are labeled, other than noticing "P" or "Probability".

Number of cars, X , owned by families

$X = x$	$P(x)$
0	.15
1	.55
2	.2
3	.1

This table shows that 15% of families own 0 cars, 55% of families own 1 car, etc. Read another way, it shows that the data point "0 cars" occurred 15% of the time, the data point "1 car" occurred 55% of the time, etc.

If you were asked to predict the number of cars some new family owned — the "expected value" for "Number of cars owned" — your prediction would just be the average of the data you have so far.

To find this average or "expected value", you would calculate:

$$0(.15) + 1(.55) + 2(.2) + 3(.1) = 1.25$$

Families own an average of 1.25 cars.

2016 AP English Language and Composition Exam Scores

Score = x	$P(x)$
5	.106
4	.175
3	.272
2	.321
1	n

1. Given that 1-5 are the only possible scores, what is the value of n ?

2. What percentage of students passed the exam with a score of 3 or higher?

3. What was the average score on the 2016 AP English Language and Composition Exam, to the nearest hundredth?

4. Based on this information, what is the expected value for the score of an unknown student taking the exam?

5. How do your answers to #3 and #4 compare?

6. [This bar graph](#) shows the number of specialized diets utilized by families of children with autism spectrum disorders (ASD) who reported following at least one specialized diet. Construct a probability distribution based on this data and find the expected value. Approximate "number of children" to the nearest ten. Email or show to me next time.

Reading: Question Types

A few question types frequently show up in ACT and SAT reading passages.

Vocabulary-in-Context Questions

These are questions like, “The word ‘champion’ in line 23 most nearly means:” Follow a simple process for answering these questions:

- Read the lines around the specified word.
- *Without looking at the answer choices*, come up with your own word to replace the one being asked about in the question.
- Match your word to the answer choices.

Correct answers to these questions will almost never be the standard, primary definition. (Hint: if you think you can answer a Vocabulary in Context question without ever looking at the passage, your answer is almost certainly wrong.) In the example above, “champion” might mean “to advocate,” but will almost certainly NOT mean “win” or “winner.”

Which of the following/All of the following except/Which of these are NOT...

Use common sense to determine when you might need to look at the answer choices in advance. If a question directly refers to the answer choices, you should obviously read them so that you can properly answer the question.

When questions are confusing — especially NOT questions — come up with a simple guideline for picking or eliminating answer choices. If the question asks, “Which of the following is NOT an argument the author would say contradicts his ideas?” then you might say to yourself, “If the author thinks it’s true, it’s the answer.” It’s much easier.

Why did the author/character...

Authors and characters are people too, with human motivations. If you’re unsure why an author or character would write or do something, ask why you yourself would do that same thing. The author or character probably had a similar reason.

UNIT 4

Adverbs and Adjectives

For the most part, you should have little difficulty in distinguishing adverb usage from adjective usage, but this is a good review just to make sure.

Adjectives are descriptive words that modify nouns. They are words that can fit in the format, "The (noun) is (adjective)," although they certainly can be used in many other ways. Orange, indistinguishable, moderate, and inky are all adjectives.

Adverbs are descriptive words that modify verbs, and they often (although not always) end in -ly. They can fit in the format, "The (noun) (verbed) (adverbly)," although they again do not have to be used exactly in this way. Examples of adverbs are quickly, mightily, almost, and mostly.

Determine whether the underlined word is used correctly; change if necessary.

1. The truck drove by very quick as if its journey were much more urgent than the idyllic setting would indicate.
2. Some documentaries are more informatively while others are more focused on entertaining.
3. Not including the books that lay strewn around the house, Elizabeth's house was decorated starkly and seemingly without any regard to warmth or humanity in general.
4. Waking up in the morning feeling like P. Diddy is an interesting request, as I don't know how to conceive of how someone as famously as P. Diddy would feel in the morning.

Comparisons

Comparisons almost always involve the word THAN. This topic is similar to parallelism in that you're looking for grammatically equivalent terms. The only real difference in a comparison is the use of "than" or similar construction in between the items. On the ACT, comparisons are tested in two ways.

Correct and Equivalent Items

You can only compare things that are actually comparable!

- | | |
|--|---|
| <p>1. I wouldn't say it, but I actually liked the job Spike's barber did more than Bob's hair.</p> | <p>4. The differences between me and you far exceed the differences that you and Bob have discovered between the two of you.</p> |
| <p>2. They usually agreed on everything, but Butch liked the green car much more than Ellen.</p> | <p>5. Joe tried in vain to make the case that, compared to the cost of the newer model, the older model was actually a very good value.</p> |
| <p>3. Never before had I seen a star that was quite as bright as the light from Sirius, which is supposed to be the brightest star in the sky.</p> | |

Proper Word Choice

-er vs. -est words, number vs. amount, etc.

Countable items: number, many, few. Non-countable items: amount, much, little

- | | |
|--|---|
| <p>6. Between ice skating and roller blading, I like roller blading most.</p> | <p>9. If one were to count the arguments in favor of each side, there would be much more in favor of expanding the program than there would be in favor of cutting its funding.</p> |
| <p>7. You can fit a higher amount of oranges in a crate than in a bag.</p> | |
| <p>8. My respect for him would be much higher if he could simply admit making mistakes that nobody even thinks are a big deal.</p> | <p>10. Worldwide wealth has increased in number at the same time that worldwide poverty has increased even more markedly.</p> |

Idiomatic Prepositions

[Grockit, UA](#)

id·i·o·mat·ic /,idēə'matik/ Adj. Using, containing, or denoting expressions that are natural to a native speaker: "distinctive *idiomatic* dialogue".

Idiomatic expressions are characteristic to a particular language, despite not necessarily making grammatical or logical sense. For example, it is idiomatic to say, "I am going to take a shower," even though you're not actually taking anything - maybe doing or experiencing, but not taking.

Idiomatic prepositions are the main category of idiomatic usage tested on the ACT. If you see a preposition underlined, the test is asking you whether the preposition chosen is the correct one based on how we state an expression in standard English.

There is no definitive list of idiomatic preposition usage, although can Google them if you find that you specifically need to study them more.

Idiomatic Prepositions Practice

Identify whether each underlined preposition is idiomatically correct.

1. The teacher tried to discourage her students in buying essays online.

4. It is important to guide our youth at the right direction whenever possible.

2. I look back fondly towards my days in high school.

5. Hazel used the confusion into her own benefit.

3. Frank went in search for an answer to his question.

6. Alonzo has been scared of dogs to the age of three.

Multiplying Quadratics and Other Polynomials[Kuta](#)

Expand using FOIL and the distributive property:

1. $x(x + 3)$

5. $(2x + 1)(3x - 2)$

2. $(x + 4)(x + 6)$

6. $(x - 8)(x - 1)$

3. $(x + 3)^2$

7. $(x + 4)(x - 4)$

4. $3x(x - 5)(x + 2)$

8. $2x^2(4x^2 - 1)$

Factoring and Solving Quadratics

[IXL 1](#), [IXL 2](#), [IXL 3](#), [IXL 4](#), [MathIsFun 1](#), [MathIsFun 2](#), [PurpleMath](#)

To solve a quadratic equation, first arrange the equation into standard form, $ax^2 + bx + c = 0$. Then, you have two possibilities:

- Factor, set each factor equal to zero, and solve to find the solutions.
- Use the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

You must always find two answers (but those two answers can be the same) to a quadratic equation. Also, factoring always begins by taking out a greatest common factor. There are a few general patterns/types of factoring that you should know. (Note that factoring by grouping, useful when $a \neq 1$, is covered separately in this binder.):

Greatest common factor:

$$x^2 + 13x = x(x + 13)$$

Difference of squares:

$$x^2 - 16 = (x + 4)(x - 4)$$

Perfect squares:

$$x^2 + 2xy + y^2 = (x + y)^2$$

$$x^2 - 2xy + y^2 = (x - y)^2$$

Find numbers that multiply to c , add to b :

$$x^2 + 4x - 21$$

$$(x - 3)(x + 7)$$

1. $7x^2 = 14x$

6. $x^3 = 25x$

2. $x^2 + 9x = -14$

7. $3x^3 - 15x = 15x^2$

3. $x^2 + 4x + 4 = 0$

8. If $x^2 + y^2 = 13$ and $xy = 5$, what is $(x - y)^2$?

4. $2x^2 - 5x = 12$

9. If $x^2 - y^2 = 10$ and $x - y = 4$, what is the value of $x + y$?

5. $x^4 - 3x^2 = 4$

Probability

[MathIsFun](#), [Kuta 1](#), [Kuta 2](#), [More](#) (with [answers](#))

The setup for probability is:

$$\frac{\# \text{ of desired outcomes}}{\# \text{ of possible outcomes}}$$

Example: If there are 8 mint cookies and 4 shortbread cookies, what is the probability that a randomly selected cookie will be mint?

$$\frac{\text{Desired outcomes}}{\text{Possible outcomes}} = \frac{8 \text{ (mint cookies, which we desire)}}{12 \text{ (total possible cookie choices)}} = \frac{8}{12} = \frac{2}{3}$$

To find the probability of multiple events, multiply the individual probabilities.

Ex. A: What is the probability that two rolls of a die will both yield numbers higher than 4?

The probability of a 5 or 6 on each roll is $\frac{2}{6}$, or $\frac{1}{3}$. The overall probability is $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

To find probability within a single event, add probabilities. The sum of the probabilities of all possible outcomes of an event must always equal 1 (= 100%).

Ex. B: The probability of a QB throwing a completion is $\frac{3}{5}$, and the probability of throwing an incomplete pass is $\frac{7}{20}$. If the only three outcomes are completion, incomplete pass, or interception, what is the probability of an interception on any given throw?

If the probability of an interception is x , then $\frac{7}{20} + \frac{3}{5} + x = 1$. Solve for $x = \frac{1}{20}$.

Combine these two rules to solve more complex questions.

Ex. C: What is the probability that this QB throws three straight non-complete passes?

The probability of a not-complete pass is $\frac{7}{20}$ (incomplete) + $\frac{1}{20}$ (interception) = $\frac{2}{5}$.

The probability of this happening three times in a row is $\frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} = \frac{8}{125}$.

When objects are physically removed after selection, decrease the numbers each time.

Ex. D: If a drawer has 6 old forks and 5 new ones, what is the probability of selecting two forks and having them both be new?

The probability for the first fork is $5/11$. The probability for the second fork is $4/10$ because one of the new forks has been removed. Multiply the two probabilities for the solution, **2/11**.

Probability Practice

5. What is the probability that a fair coin flip will yield heads 5 times in a row?
6. A bowl of candy has 5 Snickers and 7 Milky Ways. What is the probability that two randomly selected candies will both be Snickers?
7. A batter has a $\frac{1}{5}$ chance of getting a hit. What is the probability that he will get a hit in all three at bats?
8. What is the probability that this same batter will get no hits in three at bats?
9. What is the probability that this same batter will get at least one hit, but not all hits, in three at bats?
10. Three students are to be randomly selected from a group of 5 boys and 4 girls. What is the probability that all three selected students will be girls?
11. What is the probability that two randomly selected days of the same week are both weekdays?
12. If John gets pwned at WoW $\frac{3}{5}$ of the time, what is the probability that he can play three games without getting pwned once?
13. What is the probability that a three-digit number either starts or finishes with a 2?
14. What is the probability that a randomly selected card from a fair deck is either an even number or a club?

Permutations, Combinations, and One-of-Each-Group

[Kuta 1](#), [Kuta 2](#), [Kuta 3](#), [MathIsFun](#), [PurpleMath](#)

Permutations and combinations involve selecting multiple items out of one larger group. Picking one item from each of several different groups *is neither a permutation nor a combination!* On these questions, just multiply the numbers in each group.

Permutations

In how many different ways can six students be selected as class President, Vice President, and Treasurer?

Permutations = P = Positions. This is a permutation question because the items are being selected to fill specific *positions*. Mathematically this is represented by ${}_6P_3$. On your calculator, type: 6, MATH, PRB, nPr, 3, =. The answer is 120.

Combinations

In how many different ways can six students be selected for three spots on the student council?

Combinations = C = Chosen. This is a combination question because the items are either chosen or they're not; they're in or they're out. Mathematically this is represented by ${}_6C_3$. On your calculator, type: 6, MATH, PRB, nCr, 3, =. The answer is 20.

1. How many different bookshelf arrangements of four books can be made from a group of six books?
2. In how many ways can four different items from a buffet be chosen if there are seven items available?
3. How many different ways can four books be selected from a group of six?
4. If a MilkyShake is made using two different flavors of ice cream, and there are 17 different ice cream flavors available, how many different MilkyShakes are possible?
5. In how many different sequences can John call 5 of his 7 friends to invite them to a dinner party?
6. How many different routines can be made by a comedian who knows eight jokes but only has time for four?
7. Ellie can choose from 3 different appetizers, 5 different entrees, and 4 different desserts. If she must choose one item from each category, how many different ways can she fill up her plate?
8. How many different itineraries can be made if one activity per day is chosen for 3 days, out of a list of 8 possible activities?
9. Willy is a salesman, and on a certain day he is given a list of 6 clients and told that he must visit 3 of them. In how many different ways can Willy select 3 clients to visit?

Triangles in General

[IXL 1](#), [IXL 2](#), [MathIsFun](#), [Kuta](#)

Equilateral triangles: all sides are equal, and all three angles are 60° . When an equilateral triangle is cut in half, it results in two 30° - 60° - 90° triangles.

Isosceles triangles: two sides are equal, as are the angles opposite those sides.

And two facts that apply to *all triangles*:

- The sum of the three angles is always 180° .
- The two shorter sides combined must be longer than the longest side.

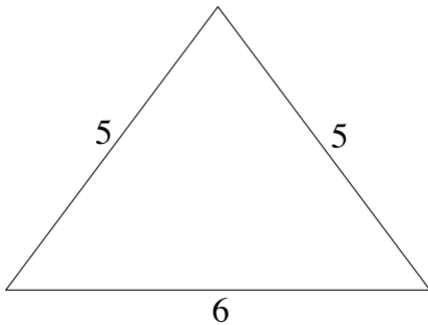
Area of a Triangle

The formula for the area of a triangle is $Area = \frac{1}{2}(base)(height)$. Height is always measured from the highest vertex of the triangle, straight down, perpendicular to the base.

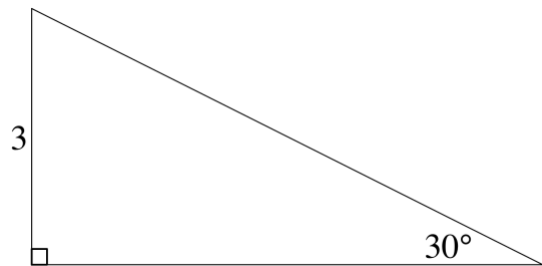
Triangle Practice Questions

Find the area of each triangle.

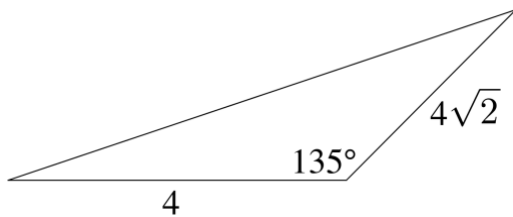
1.



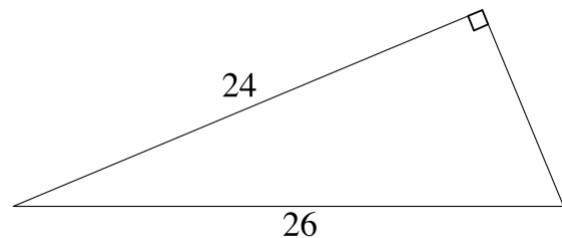
3.



2.



4.



5. The angles of a triangle are in the ratio of 3:4:8. What is the measure of the largest angle?

6. Two sides of a triangle are 8 and 10. If the third side is an integer length, what are the shortest and longest possible lengths of the third side?

7. $\triangle ABC$ is isosceles and side AB is 10 units long. If its perimeter is 32 units, what are the possible lengths of side AC ?

- (A) 10 and 12
- (B) 10, 11 and 12
- (C) 11 and 12
- (D) 12 only
- (E) Any length between 10 and 12

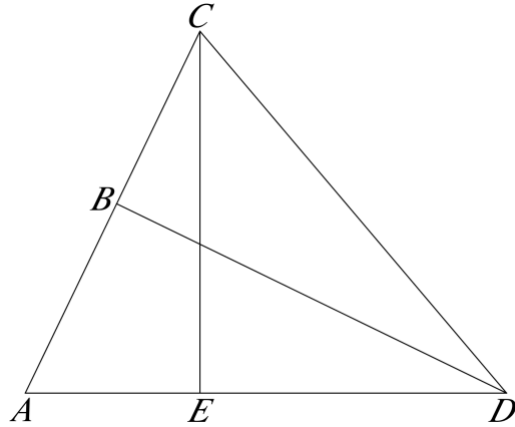
8. One side of an equilateral triangle is 6 units long. What is the triangle's area?

9. Which of the following is could not be the lengths of the sides of a triangle?

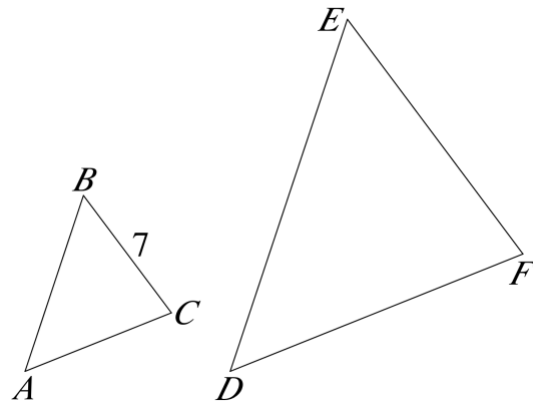
- (A) 1, 1, 1
- (B) 0.11, 0.21, 0.31
- (C) 22, 44, 66
- (D) 4.2, 8.8, 12.0
- (E) 0.03, 0.04, 0.05

10. The base of an isosceles triangle is 12 units, and the other two sides are 10 units. What is the area of the triangle?

11. A rectangle measures 60 feet by 80 feet. What is the longest possible distance between two points on this rectangle?



12. In the triangle above, $AD \perp CE$ and $AC \perp BD$. Additionally, $AD = 12$, $CE = 9$, and $AC = 10$. What is the length of BD ?



13. In the diagram above, $\triangle ABC$ is similar to $\triangle DEF$ and $BC = 7$. If the perimeter of $\triangle ABC$ is 20 and the perimeter of $\triangle DEF$ is 30, what is the length of \overline{EF} ?

Rectangles

$$\text{Area} = L \times W$$

$$\text{Perimeter} = 2L + 2W$$

1. The perimeter of a rectangle is 56 and one of its sides is 11. What is its area?
2. The length of a rectangle is 3 less than twice its width. The perimeter of the rectangle is 36. What is its area?

3. A rectangle with sides in a ratio of 3: 1 is fully contained inside another rectangle whose sides are in the same ratio but each of whose legs is 1.5 times as long. What is the probability that a point in the large rectangle is also in the small rectangle?

Squares

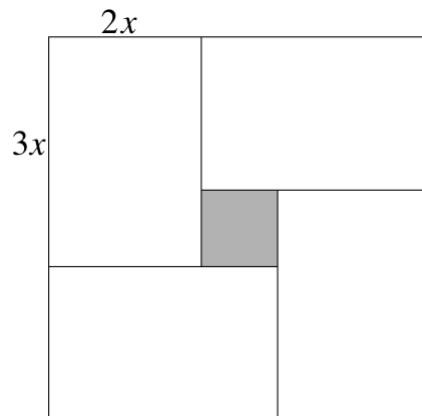
$$\text{Area} = s^2$$

$$\text{Perimeter} = 4s$$

$$\text{Diagonal} = s\sqrt{2}$$

4. The diagonal of a square is 10 units. What is the square's perimeter?
5. The perimeter of a square is 14 centimeters. What is the area of the square, in square centimeters?
6. The perimeter of a square is 28. What is the square's area?
7. A square has sides of length x , and a rectangle has width y and a length twice that of the square. If the rectangle has a larger area, what is the difference in area between the rectangle and the square?
 - (A) $y^2 - x^2$
 - (B) $2y - x$
 - (C) $xy - x^2$
 - (D) $x(2y - x)$
 - (E) $y(2x - y)$

8. Will is putting up a fence to completely enclose a square plot of land with an area of 3,025 square yards. If fencing is sold only in 15-yard lengths, how many yards of fence must Will purchase?

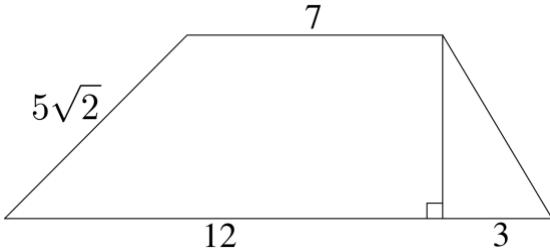


9. In the figure above, the four unshaded rectangles are congruent. If $x = 4$, what is the area of the shaded region in the center of the figure?

Other Quadrilaterals

Trapezoids

Area = (average of the bases) • (height)

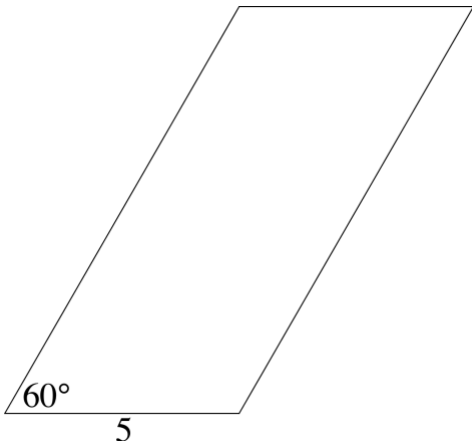


1. What is the area of the trapezoid

Parallelograms

Area = base • height

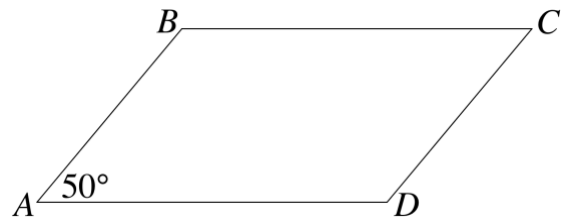
3. What is the area of a parallelogram with sides of length 8 and 12, and with one interior angle of 45° ?



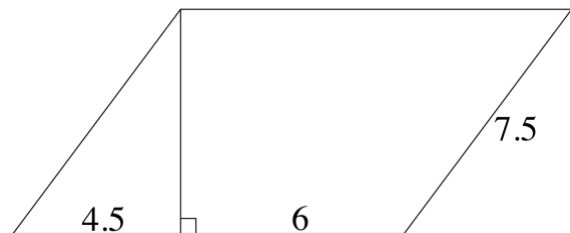
4. A parallelogram has a base of 5 inches and an interior angle of 60° , as shown above. Its area is $15\sqrt{3}$ square inches. What is the length of one of the sides, in inches?

shown above?

2. An isosceles trapezoid has bases of 13 and 37, and sides of 20 units each. What is the trapezoid's area?



5. In the diagram above, $ABCD$ is a parallelogram. What is the measure of angle B ?



6. What is this parallelogram's area?

7. Which of the following statements guarantees that quadrilateral $ABCD$ MUST be a parallelogram?

- I. Angles B and C add up to 180° .
- II. The quadrilateral is a rectangle.
- III. The quadrilateral contains two pairs of congruent angles.

Averages[MathWarehouse](#), [OML](#)

ALL averages are calculated using the same formula

$$\text{Average} = \frac{\text{Sum of Quantities}}{\text{Number of Items}} \quad \left(A = \frac{\text{Sum}}{\text{Number}} \quad A = \frac{S}{N} \quad S = A \times N \right)$$

Average Practice Questions:

1. The average of 3, 7, 10, and x is 8. What is x ?
2. What is the average of $x + 3$, $5x - 4$, and $3x + 7$?
3. The average of x and y is zero. What is the ratio of $3x$ to $2y$?
4. In a group of five students, four of the students have a total of \$37. How much would the fifth student need to have in order for each student in the group to have an average of \$12?
5. What is a if the average of $a + 6$, $4a - 1$, $4a + 7$, and $7a$ is -9 ?
6. What is the average of 22, $3x + 1$, and 13?

Mixtures and Complex Averages

[SoftSchools](#), [MathWarehouse](#), [OML](#)

More difficult average problems on the usually involve combining averages, concentrations, rates, percentages, etc. for two different groups into one combined, overall group.

NEVER take the average of two averages!

Instead, use the mixture formula, which works for all questions of this type. n represents the number or quantity of items, and A represents those items' average, concentration, etc.:

$$n_1A_1 + n_2A_2 \dots = n_{Total}A_{Total}$$

Examples:

A student averages 81% on the first two quizzes. What does the student need to average on the next six quizzes to bring his quiz average to 90%?

$$2(81) + 6x = 8(90)$$

Solve for $x = 93$

What quantity of a 0.05M solution must be added to 0.3L of a 0.5M solution to produce a 0.18M solution?

$$x(.05) + .3(.5) = (x + .3)(.18)$$

Solve for $x \approx 0.74L$

- Jeremy drove for two hours at 60 mph, and then for three hours at 70 mph. What was his overall average speed?
- Five football players weigh an average of 240 pounds. If the two lightest, who average 225 pounds, are removed, what is the average weight of the remaining three?
- Eliza had a sales-to-appointments ratio of 0.07 in September. In October she made twice as many calls but her ratio dropped to 0.04. What was her combined ratio for the two months?
- In a certain company, 10% of the employees are executives. If the average executive salary is \$195,000 and the average salary for the rest of the company is \$50,000, what is the overall average salary for the company?

5. Each student in a group has an average of \$12. If two students with no money join, the average goes down to \$8. How many students were in the group before the two others joined?
6. A quarterback completes 52% of his passes in his first two games. What completion percentage would he need in the next six games to average 70% overall?
7. Patrick bowled five games, averaging 191. For how many games would he need to average 215 to raise his overall average score to 200?
8. Charise must average 8 sales each month over the course of a year to earn a bonus. Through 5 months she averaged 11 sales per month. For the next 4 months she average only 4.5 sales per month. What must she average over the final 3 months to earn her bonus?
9. In Adi's math class, homework is worth 20% of a student's overall grade, quizzes are worth 30%, and tests are worth 50%. Adi's average homework score is 95%, and her average quiz score is 80%. What must she average on her tests to earn an overall grade of exactly 80%?
10. During a certain Monday through Friday period, the temperature each day was higher than it was the temperature the day before. Which of the following statements **MUST** be true?
- (A) There was a mode temperature for the week.
- (B) Wednesday's temperature was the average (arithmetic mean) temperature for the week.
- (C) Wednesday's temperature was the median temperature for the week.
- (D) The temperature increase from Monday to Wednesday equaled the temperature increase from Wednesday to Friday.
- (E) Tuesday's temperature was the average of Monday and Wednesday's temperatures.

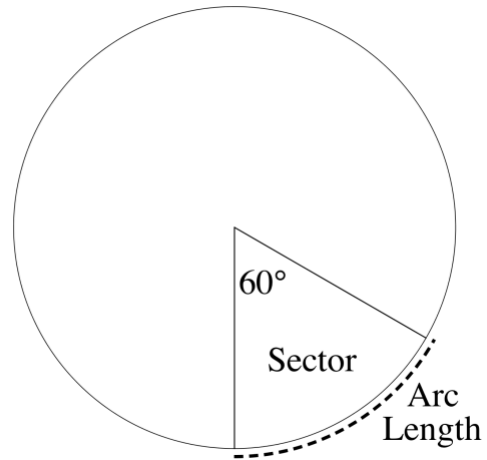
Circles: Sectors

[MathIsFun](#), [Kuta](#), [PurpleMath](#), [IXL 1](#), [IXL 2](#)

Sectors, arc lengths, and central angles of circles are always proportional to each other.

For example, in the circle at right:

The arc is $\frac{1}{6}$ of the circumference;
 The central angle is $\frac{1}{6}$ of 360° ; and
 The sector is $\frac{1}{6}$ of the area of the circle.



When a question involves sectors, arc lengths, and central angles, figure out what “fraction” of the circle the sector takes up — in any of these three ways — and go from there.

- The area of a circle is 16π , and the angle measure of a sector is 30° . What is the area of that sector?
- The radius of a circle is 3 and the area of a sector in that circle is 3π . What is the sector's angle measure?
- What is the angle measure of a sector whose arc length is 4π in a circle whose area is 100π ?
- A sector has a central angle of 15° , and the circle has a radius of 6. What is the arc length of the sector?
- The hour hand of a clock is 4 inches long. How many inches does the tip of this hour hand travel from 9:00 a.m. to noon on the same day?
- The minute hand of a clock is 2.5 cm long. How far does the tip of this minute hand travel from 10:45 a.m. to 2:30 p.m.?

All About Quadratics

A parabola results from graphing any quadratic equation of the form — in other words, $f(x) = x^2$. The graphs of $y = x^2$, $f(x) = x^2 + 4x - 7$, and $g(c) = c^2 + 12$ are all parabolas. All parabolas are symmetrical on either side of the vertex.

The coefficient of the quadratic term, a , tells you about the direction and amplitude (steepness) of the parabola. A parabola starting $y = x^2$ will open upwards; a parabola starting $y = -x^2$ will open downwards. A parabola with a coefficient $|a| > 1$ will be steep and appear narrow; a parabola with a coefficient $|a| < 1$ will be flattened and appear squat.

Parabolas can be written in three different formats:

Standard Form: $f(x) = ax^2 + bx + c$

To get it: Multiply out one of the other forms.
You can immediately see: the y -intercept. It's always $(0, c)$.
You can figure out: the x -coordinate of the vertex. It's $x = -\frac{b}{2a}$
 the y -coordinate of the vertex. Plug that x into the function.
 the sum of the zeroes, $-\frac{b}{a}$
 the zeroes, using the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Factored Form: $f(x) = a(x - c)(x - d)$

To get it: Get to standard form, then factor.
You can immediately see: the zeroes, c and d . Set each factor equal to zero and solve.
You can figure out: the x -coordinate of the vertex by averaging the zeroes, $x = \frac{c+d}{2}$
 the y -coordinate of the vertex. Plug that x into the function.

Vertex Form: $f(x) = a(x - h)^2 + k$

To get it: Get to standard form, then complete the square or find vertex.
You can immediately see: the vertex — i.e. the maximum or minimum value of the function and where it occurs. It's at (h, k) .
You can figure out: not much else without converting to standard form.

Roots and Zeroes

A quadratic function can have two, one, or no real zeroes:

- If the graph never touches the x -axis, the function has no real zeroes (but 2 complex zeroes).
- If the graph is tangent to the x -axis, it has one real zero (a.k.a. a "double root").
- If the graph crosses the x -axis twice, it has two real zeroes.

UNIT 5

Radians

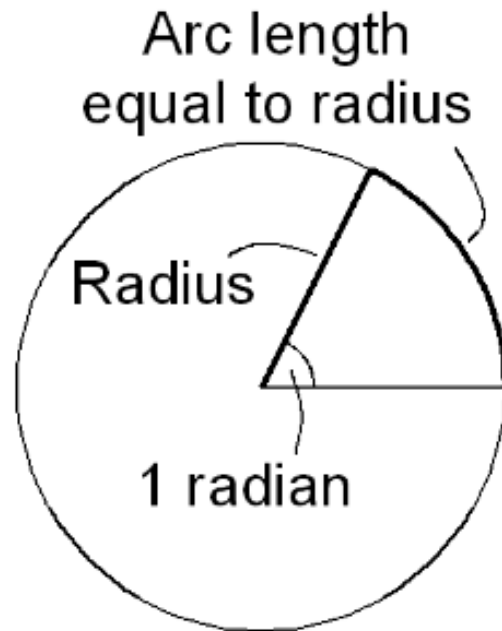
[Online, MathIsFun](#)

Radians are another way of measuring angles. They do the same thing as degrees; they're just a different size. It's no different than inches and meters or seconds and hours.

Radians do not have a symbol. You will recognize an angle is being measured in radians by the absence of a degree symbol! An angle labeled 10° is measured in degrees; an angle labeled just 10 is measured in radians.

One *radian* is the angle formed from the middle of a circle by an arc length equal to the radius. It is approximately 57° .

We know how many radii it takes to go around a circle because of the formula $C = 2\pi r$. All the way around a circle is 360° ; therefore $360^\circ = 2\pi r$, so $\pi r = 180^\circ$. This is the fundamental fact to memorize about radians: **π radians is 180° !**



How do you convert between degrees and radians?

To convert between degrees and radians, use dimensional analysis with $\pi = 180^\circ$.

Ex. A: Convert 120° into radians.

$$\frac{120^\circ}{1} \times \left(\frac{\pi}{180^\circ} \right) =$$

The degree symbols cancel out and the fraction simplifies to $\frac{2\pi}{3}$.

1. Convert 45° to radians.

2. Convert $\frac{\pi}{6}$ to degrees.

Ex. B: Convert $\frac{7\pi}{4}$ to degrees.

$$\frac{7\pi}{4} \times \left(\frac{180}{\pi} \right) =$$

The π 's cancel out and the resulting numbers simplify to 315° .

3. Convert 210° to radians.

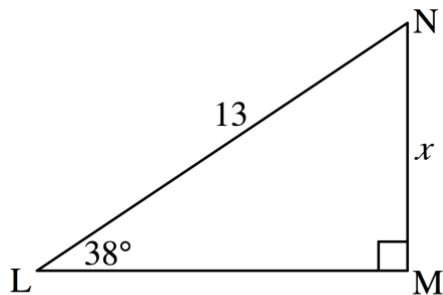
4. Convert $\frac{5\pi}{3}$ to degrees.

Trigonometric Functions

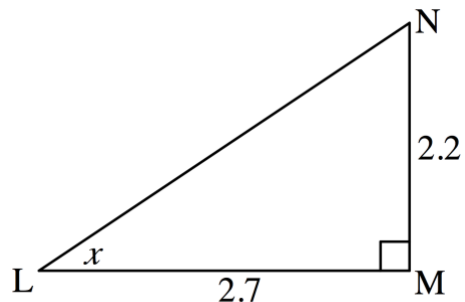
[IXL 1](#), [IXL 2](#), [MathIsFun 1](#), [MathIsFun 2](#), [MathIsFun 3](#), [MathIsFun 4](#)

Complete all questions involving trig functions using the same process:

- Select your reference angle.
- Identify the two sides of interest *relative to your reference angle*.
- Choose the appropriate trig function to match the angle and sides you chose.
- Set up the equation as $\text{function}(\text{angle}) = \frac{\text{side}}{\text{side}}$
- Solve by isolating the variable.



Ex. A: Based the diagram above, write a trigonometric expression for the length of side BC, indicated by x .



Ex. A: Based the diagram above, write a trigonometric expression for the measure of angle L , indicated by x .

Following the steps listed above:

- Our reference angle is angle A, 38° .
- Based on angle A, we're interested in finding x , which is the opposite side; the only side we have information about is AB, which is the hypotenuse.
- We're using the opposite side and the hypotenuse, so the appropriate trig function is sine (SOH).
- The equation is $\sin(38^\circ) = \frac{x}{13}$
- We multiply both sides by 13 to obtain $x = 13 \sin(38^\circ)$, our final answer.

Following the steps listed above:

- Our reference angle is angle L; we don't know its value.
- Based on angle L, we know the opposite side, which is 2.2, and the adjacent side, which is 2.7.
- We're using the opposite side and the adjacent side, so the appropriate trig function is tangent (TOA).
- The equation is $\tan(x) = \frac{2.2}{2.7}$
- Apply \tan^{-1} to both sides to "undo" tangent and isolate x , obtaining $x = \tan^{-1}\left(\frac{2.2}{2.7}\right)$, our final answer.

One more good thing to know: $\sin(x) = \cos(90^\circ - x)$, and $\cos(x) = \sin(90^\circ - x)$.

Terminology

	<i>Guess</i>		<i>Actual</i>
the sum of x and y is 12	_____	sum	_____
how many boats are there?	_____	how many	_____
the distance between 2 and 14	_____	distance	_____
four times the quantity $2x + 3$	_____	the quantity	_____
m is greater than n	_____	is greater than	_____
the ratio of dogs to cats	_____	ratio	_____
6 less than 11 is 5	_____	less than	_____
the difference between a and b	_____	difference	_____
John runs 3 times per week	_____	per	_____
7 boxes of a dozen eggs	_____	of	_____
25 percent of 16 is 4	_____	percent or %	_____
the opposite of x is 5	_____	opposite	_____
x is three times as big as r	_____	is	_____
4 out of 5 dentists prefer Crest	_____	out of	_____
what is 2 plus 2?	_____	what	_____
x is less than 14	_____	is less than	_____
the product of two integers	_____	product	_____
a is 7 more than twice b	_____	more than	_____
all except 3 were boys	_____	except	_____
the reciprocal of x	_____	reciprocal	_____

Absolute Value

[MathIsFun](#), [PurpleMath 1](#), [PurpleMath 2](#), [Kuta 1](#), [Kuta 2](#)

Absolute value equations are solved in the following steps:

1. Isolate the absolute value.
2. Set up two equations - one with a positive answer and one with a negative answer.
3. Solve each equation to determine your two answers.

$$3|4x - 1| + 2 = 11$$

$$3|4x - 1| = 9$$

$$|4x - 1| = 3$$

$$4x - 1 = 3 \quad \text{and} \quad 4x - 1 = -3$$

$$4x = 4 \quad \text{and} \quad 4x = -2$$

$$x = 1 \quad \text{and} \quad x = -\frac{1}{2}$$

1. $|x - 4| = 6$

2. $2 + 3|2x + 1| = 23$

3. $-2|x + 7| = -12$

4. $4|x - 3| + 6 = 10$

5. $1 + 2|3x + 4| = 5$

6. $5 + 4|3x - 2| = 1$

7. Which of the following equations has the least number of real solutions?

(A) $x^2 = 12$ (B) $x^3 = 12$

(C) $x^4 = 1$ (D) $|x| = 7$

(E) $|x - 4| = 7$

8. Which of the following equations has no solution?

(A) $|x - 2| = 8$ (B) $|x - 8| = 2$

(C) $|x| + 2 = 8$ (D) $|x| + 8 = 8$

(E) $|x| + 8 = 2$

9. In which of the following equations could $x = 5$?

(A) $|x| = 6$ (B) $|x - 5| = 6$

(C) $|x - 5| = 11$ (D) $|x - 6| = 5$

(E) $|x - 8| = 3$

Absolute Value Inequalities

[PurpleMath](#), [Regents](#), [SOS](#)

The steps for solving absolute value inequalities are as follows:

1. Isolate the absolute value.
2. Set up two inequalities. The first is the same as the original inequality without the absolute value signs. The second reverses both the inequality sign and the positive/negative sign of the answer:

$$\begin{array}{l}
 1 + 3|2x - 1| < 7 \\
 3|2x - 1| < 6 \\
 |2x - 1| < 2
 \end{array}
 \qquad
 \begin{array}{l}
 2x - 1 < 2 \quad \text{and} \quad 2x - 1 > -2 \\
 2x < 3 \quad \text{and} \quad 2x > -1 \\
 x < \frac{3}{2} \quad \text{and} \quad x > -\frac{1}{2}
 \end{array}$$

The $<$ and \leq signs produce graphs that look like barbells, as above. The $>$ and \geq signs produce rays going off in opposite directions, with a gap in between.

Solve and graph:

1. $4|x - 3| < 6$

2. $-2|2x - 1| \geq -12$

3. $3 + 2|x + 7| > 20$

4. $|x + 4| - 7 \geq 10$

5. Which of the following expressions includes all of the points on a number line whose distance from -7 is less than 5?

- (A) $|x - 5| < -7$ (B) $|x - 5| < 7$
 (C) $|x + 5| < 7$ (D) $|x - 7| < 5$
 (E) $|x + 7| < 5$

6. If $-\frac{2}{3}|-2x + 3| \leq -10$, what are the possible values of x ?

7. Which of the following statements guarantees that the absolute value of a number m MUST be less than 10?

- I. $m < 10$ II. $m > -3$ III. $|m| > -3$

- (A) I only (B) I and III only
 (C) II only (D) I, II and III
 (E) None of the statements

8. Given the inequality $-2x - 11 < -3$, which of the following numbers is NOT a possible value for x ?

- (A) -4 (B) -3 (C) -2
 (D) -1 (E) All of A-D are possible

Sequences

[MathIsFun](#), [PurpleMath](#), [Regents](#), [Kuta 1](#), [Kuta 2](#), [Kuta 3](#), [Kuta 4](#)

Arithmetic Sequences: find the common difference d , where $d = \frac{\text{difference of 2 terms}}{2\text{nd term \#} - 1\text{st term \#}}$.

1. The first term in an arithmetic sequence is -7, and the fifth term is 13. What is the seventh term?

2. If the 7th term of an arithmetic sequence is 34 and the 9th term is 39, what is the first term?

3. Billy decides to start doing push-ups every day, and to do 3 more push-ups every day than he did the day before. If Billy does 13 push-ups on day 1, on what number day would he do 200 or more push-ups for the first time?

Geometric Sequences: find the common factor x , using $(\text{term \#}a)x^{b-a} = (\text{term \#}b)$

4. The first term of a geometric sequence is 2 and the fourth term is 54. What is the fifth term?

6. The second term of a geometric sequence is 27, and the seventh term is $-\frac{1}{9}$. By what must the eighth term in the sequence be multiplied to get the tenth term?

5. The first term in a sequence is $5\frac{1}{3}$. Each subsequent term is 50% greater than the one before it. After $5\frac{1}{3}$, what is the next term in this sequence that is NOT an integer?

7. The 4th term of a geometric sequence is $\frac{1}{4}$ and the 7th term is $\frac{1}{32}$. What is the first term?

Patterns: Go forwards or go backwards, but don't take shortcuts.

8. The next term in a sequence is obtained by multiplying by 3 and then subtracting 1. If the first term is 2 and the sixth term is 365, what is the fifth term?

9. What is the next term in the following sequence: $\frac{1}{3}, 1, 4, 12, 15\dots$

Repeating Sequences

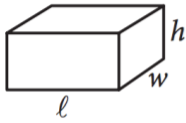
[Sophia](#), [Random](#)

- Find the *period*.
 - Divide by period, find remainder.
 - Count out to remainder's place.
1. If January 1 is a Tuesday, what day of the week is the 101st day of the year?
2. In which of the following numbers of hours will it be the same time as it will be three hours from now?
- (A) 266 (B) 267 (C) 268
(D) 269 (E) 270
3. What is the 55th term of the repeating decimal .17321732...?
4. If a bracelet is made with numbered beads in the repeating pattern green, yellow, green, blue, yellow, which of the following number beads will be green?
- (A) 20 (B) 22 (C) 25 (D) 26 (E) 27
5. A child begins singing the alphabet song over and over. What is the 1,000th letter this child will sing?
6. A golfer eagled the 13th hole during a round of golf. If his next eagle, several 18-hole rounds later, came on the 6th hole, which of the following is a possible number of holes the golfer went between the two eagles?
- (A) 131 (B) 133 (C) 135
(D) 137 (E) 139
7. What is the units (ones) digit of 2^{47} ?
8. What is the units (ones) digit of 3^{92} ?
9. A cruise ship worker's repeating schedule is 10 workdays on the ship followed by 3 days off. If today is the worker's first day off, which of the following numbers of days after today is a workday?
- (A) 130 (B) 131 (C) 132
(D) 133 (E) 134

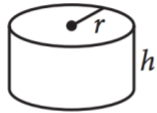
Volume

[IXL 1](#), [IXL 2](#), [IXL 3](#), [MathIsFun](#), [Kuta 1](#), [Kuta 2](#), [More Spheres](#)

The test will provide you with the following formulas (which ideally you will memorize):



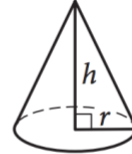
$$V = \ell wh$$



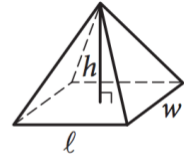
$$V = \pi r^2 h$$



$$V = \frac{4}{3} \pi r^3$$



$$V = \frac{1}{3} \pi r^2 h$$



$$V = \frac{1}{3} \ell wh$$

You should also memorize that the volume of a cube is $V = s^3$. As with any other formulas, utilize volume formulas by plugging in what you know and solving for what you don't.

1. A shipping container has square ends. The container is 9 feet tall and has a volume of 3,078 cubic feet. What is the length of the container?

2. A cylindrical tank has a diameter of 10 meters and is 6 meters high. What is the tank's volume?

3. The volume of a sphere is 36π cubic inches. What is its radius?

4. The diameter of the base of a cone is 16 inches. If its height is 20 inches, what is its volume?

5. The volume of a certain rectangular pyramid needs to be at least 75 m^3 . Its height is limited to 5 meters and its length to 7 meters. What is the shortest integer width that would allow the pyramid to meet the volume requirement?

6. A circular, flat-bottomed swimming pool is filled to a depth of 6 feet. If it then contains 300π cubic feet of water, what is the circumference of the pool, rounded to the nearest foot?

7. A basketball is 9 inches in diameter. What is its volume in cubic feet?

8. A cone has a volume of 15π cubic inches and a diameter of 6 inches. Another cone has the same height and a diameter of 8 inches. What is its volume?

Relationships Between Variables

[MathIsFun 1](#), [MathIsFun 2](#), [IXL 1](#), [IXL 2](#), [IXL 3](#), [Linear vs. Exponential](#)

How can we describe the relationship between two variables?

- A **correlation** or **association** means that there is *some* connection between the movement of two variables or quantities. They could go up and down together, or they could go in opposite directions. Basically, if you can use one to predict the other, there is a *correlation* or *association*. This does **NOT** mean that one causes the other! A **causal** relationship can generally only be shown through a controlled experiment.

- **Linear, Quadratic, and Exponential Relationships**

A **linear** relationship means that there is a constant rate of change between two variables. On a graph, the *line of best fit* for a linear relationship is a straight line.

The *line of best fit* for a quadratic or exponential relationship is always a curve. A **quadratic** relationship means that one quantity changes based on the *square* of another: for example, the distance an object has free-fallen depends on the square of how long it's been falling. In an **exponential** relationship, one quantity changes based on a *power* of another: for example, 1.05^2 , then 1.05^3 , 1.05^4 , and so on.

- **Direct vs. Indirect/Inverse Relationships**

A **direct** relationship means that as one quantity increases, the other increases. An **indirect** or **inverse** relationship means that as one quantity increases, the other decreases. (More about this in [Ratios and Proportions](#).)

- **Strong vs. Weak Relationships**

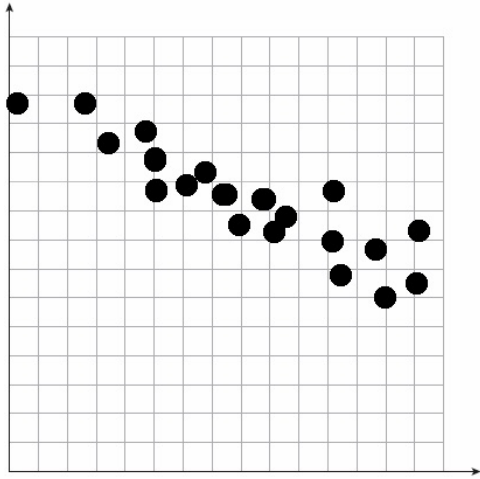
A **strong** correlation, association, or relationship means that the points on a graph form a very tight line. This means that one variable is a great predictor for the other. A **weak** correlation, association, or relationship means that the points on a graph form more of a general trend or cloud than an exact line. This means that one variable is just an okay predictor for the other — better than nothing, but not great.

- **Correlation Coefficients**

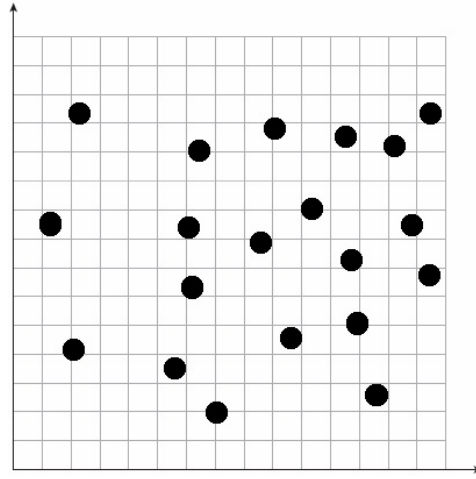
Values of correlation coefficients, r , range from -1 to 1 . Correlation coefficients show the *direction* and *strength* of relationships between two quantities as follows:

- **Positive** numbers indicate *direct relationships*.
- **Negative** numbers indicate *indirect/inverse relationships*.
- **Greater absolute value** (closer to -1 or 1) indicates a *strong relationship*.
- **Lower absolute value** (closer to 0) indicates a *weak relationship*.
- **Zero** indicates *no relationship* between two variables.

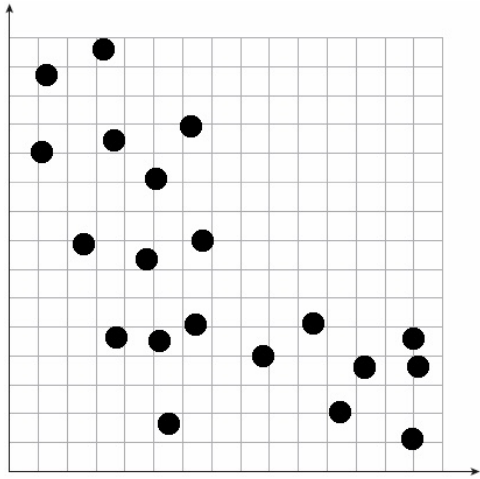
Describe each of the following relationships by their type, direction, and strength. Give an estimated correlation coefficient, r , for each relationship



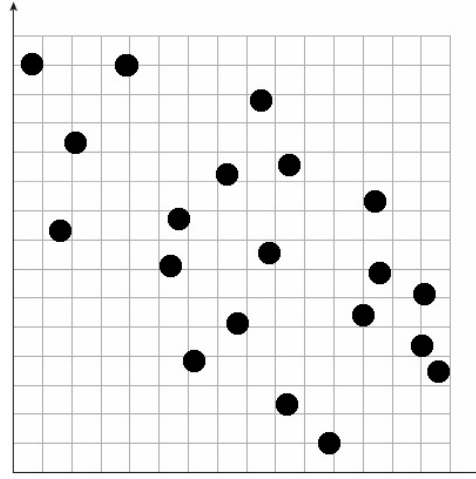
1. L/E D/I S/W $r \approx$



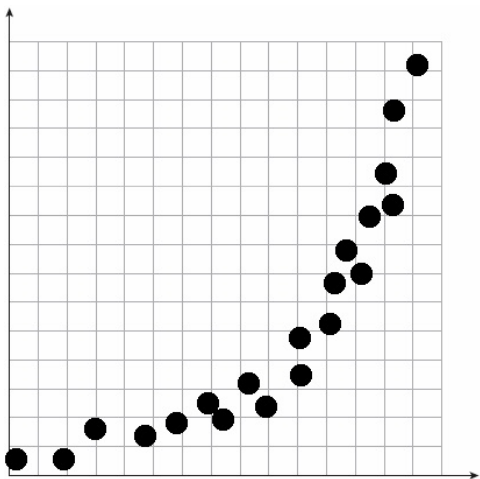
4. L/E D/I S/W $r \approx$



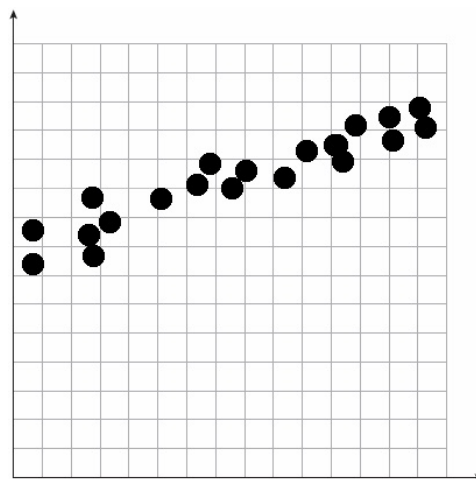
2. L/E D/I S/W $r \approx$



5. L/E D/I S/W $r \approx$



3. L/E D/I S/W $r \approx$

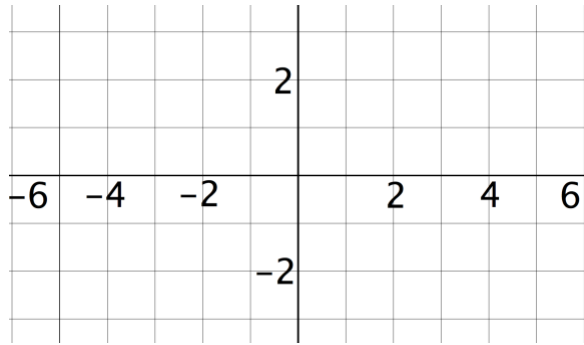


6. L/E D/I S/W $r \approx$

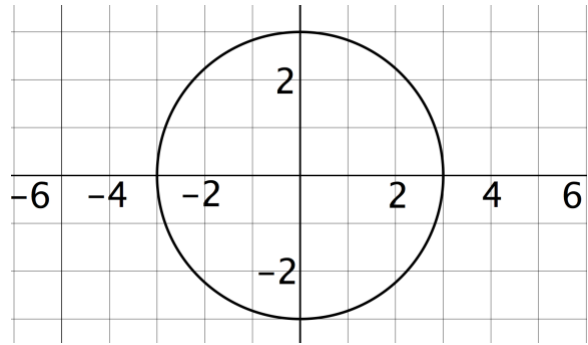
Circles: EquationsIXL, [Math Warehouse](#), [MathsFun](#)

The equation of a circle with radius r centered at (h, k) is: $(x - h)^2 + (y - k)^2 = r^2$.

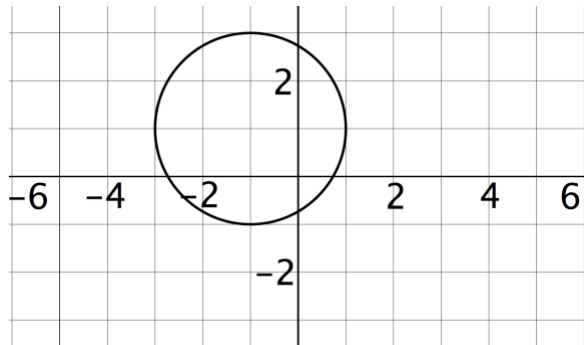
You won't have to do anything complicated with this equation*. You'll either be given a graph and asked which equation fits, or given an equation and asked which graph fits.



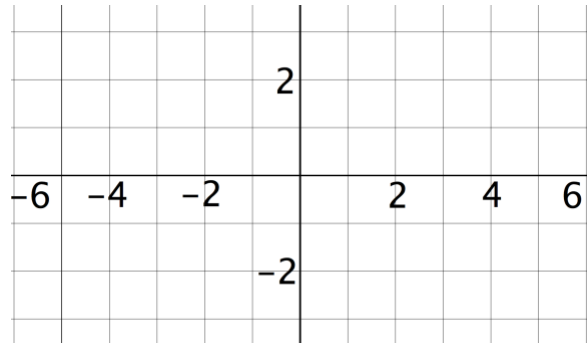
1. Draw the graph of the equation $(x - 2)^2 + (y + 1)^2 = 6.25$.



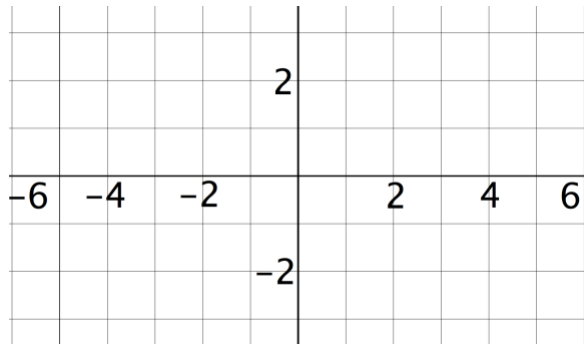
4. Write the equation of the graph shown in the figure above



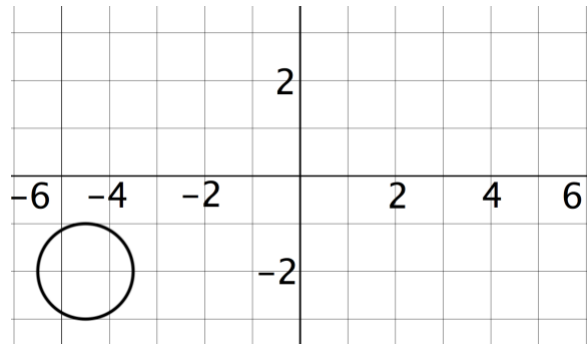
2. Write the equation of the graph shown in the figure above.



5. Draw the graph of the equation $(x - 1.5)^2 + (y + 1.5)^2 = 2.25$.



3. Draw the graph of the equation $x^2 + (y - 1.5)^2 = 4$.



6. Write the equation of the graph shown in the figure above.

Word Problem Practice Set

- Two more than twice a number is the same as seven less than three times that number. What is the number?
- The positive difference between two numbers is one more than twice their sum. The larger number is nine more than double the smaller number. What is the smaller number?
- Billy is four years older than Alex. Charlie is twice as old as Billy. In three years, Charlie will be three times as old as Alex. How old is each person now?
- Four less than a number is the same as double that number. What is the number?
- The ratio of $4x$ to seven is equivalent to the ratio of $3y$ to 6. What is the ratio of x to y ?
- If $(x + h)(x + k) = hk$ and $x \neq 0$, what is the sum of x , h and k ?
- When the digits of the two-digit number AB is reversed, the result is the two-digit number BA . If $AB + BA = 132$, what is the sum of $A + B$?
- A certain number is a multiple of 490 and has exactly 4 distinct prime factors. What is one possible value for this number?
- When x is divided by 11, the remainder is 8. What is the remainder when $5x$ is divided by 11?
- A sphere has a radius of $3\sqrt{3}$. What is the volume of the largest cube that can fit entirely inside this sphere?

More Trigonometry

- Sec, Csc, Cot: sine and cosine each go with the one that they don't sound like.
 - $\sin^2 + \cos^2 = 1$
 - The meaning of \sin^{-1} and \arcsin , \csc^{-1} , etc.
 - What the graphs of \sin , \cos , and \tan look like. Remember graph-shifting rules.
 - Everything else can be solved by substituting O, H, and A (or a, b, and c) and manipulating.
1. Simplify: $1 + \cot^2 x$
 2. Find all six trig functions of the smallest angle of a right triangle whose two legs are 10 and 24. Do not use a calculator. Then, use a calculator to find the angle measure in degrees and radians.
 3. A 12-foot ladder leaning against a wall forms a 21° angle with the wall. To the nearest tenth, how many feet high up the wall is the top of the ladder?
 4. The cosine of an angle is 0.6. What is the sine of the angle? Do by hand and by calculator.
 5. The hypotenuse of a right triangle is 6.5 cm. The sine of one of the angles is 0.22. Find the measure of both angles in radians (to 2 decimal places) and the length of the other two sides (to one decimal place).
 6. Geraldo and Paco want to determine the angle of elevation of the sun at a particular time of day. Geraldo notes that Paco is exactly 6 feet tall, and measures that Paco's shadow is 2.5 feet in length. What is the angle of elevation of the sun to the nearest degree?
 7. In triangle GHK , $\sin K = \frac{\sqrt{5}}{5}$. What is $\cos K$?

Direct and Inverse Variation/Proportion

Direct variation or direct proportion means that two quantities always have the same ratio. Their quotient is always the same. You solve these questions with proportions: $Quotient_1 = Quotient_2$, the same as with regular ratio questions. There is no difference.

Example: Two quantities k and m vary directly with each other. When k is doubled, m increases by 3. What was the original value of m if $m \neq k \neq 0$?

Because these two items vary directly, their quotient is always the same and we can solve this question using a proportion. The original values will always have the same quotient as the new values:

$$\frac{k}{m} = \frac{2k}{m + 3}$$

Cross-multiply to get $k(m + 3) = 2km$; simplify to $km + 3k = 2km$; subtract to get $km = 3k$; and solve for $m = 3$ (because $k \neq 0$).

Inverse variation or inverse proportion means that two quantities always have the same product. If one quantity increases, the other one decreases in sync so that their product remains the same. To solve these questions, use the setup $Product_1 = Product_2$.

Example: Two quantities a and b are inversely proportional. When $a = 3$, $b = 25$. What is a when $b = 5$?

Because these quantities are inversely proportional, their product is always the same. We will set up $Product_1 = Product_2$:

$$\begin{aligned} (3)(25) &= (a)(15) \\ 75 &= 15a \\ a &= 5 \end{aligned}$$

To summarize:

Set up *direct proportions and direct variation* as proportions, $Quotient_1 = Quotient_2$.

Set up *inverse proportions and inverse variation* as multiplication, $Product_1 = Product_2$.

Direct/Inverse Variation/Proportion Practice

1. Two numbers m and n are inversely proportional. If $m = 4$ when $n = 20$, what is n when $m = 5$?

2. Two numbers m and n are inversely proportional. If $m = 8$ when $n = -5$, what is n when $m = -4$?

3. Two quantities A and B are directly proportional. If A is increased from 4 to 16, what happens to B ?

- (A) It increases by 12.
- (B) It decreases by 12.
- (C) It is multiplied by 4.
- (D) It is divided by 4.
- (E) It is squared.

4. Two numbers a and b vary directly with each other. When $a = 4$, $b = -6$. What is b when $a = -2$?

5. Two numbers a and b vary directly with each other. When $a = 12$, $b = 4$. What is a when $b = 6$?

6. Two numbers c and d vary directly with each other. When c is increased by 4, what happens to d ?

- (A) d decreases by 4
- (B) d stays the same
- (C) d increases by 4
- (D) d increases by 8
- (E) Cannot be determined from the given information

7. Which of the following are examples of inverse variation?

- I. Speed and time (distance constant)
- II. Speed and distance (time constant)
- III. The length and width of a rectangle (area constant)

8. The number of bacteria in a certain colony varies directly with how far the temperature is above 72° Fahrenheit. When the temperature is 78° , there are 15,000 bacteria. How many bacteria are there when the temperature is 90° ?

9. In which of the equations below are x and y directly proportional to each other?

I. $x = 4y$ II. $\frac{x}{y} = 12$ III. $y = \frac{11}{x}$

10. Two numbers x and y are inversely proportional. When x increases by 3, y increases by 2. What is the value of the sum $2x + 3y$?

11. Two quantities a and b vary directly with each other, and $a \neq b \neq 0$. When a is divided by 3, b increases by 10. What was the original value of b in this case?

12. Two quantities j and k vary directly with each other. When j is quadrupled, the value of k increases. Which of the following is a possible value of k ?

- I. -5 II. 0 III. 3.5

Complex Numbers

[MathIsFun 1](#), [MathIsFun 2](#), [PurpleMath](#), [Kuta](#), [Regents](#),

Complex numbers are those which incorporate the quantity i . The label i stands for *imaginary*, because i represents something that does not exist: $\sqrt{-1}$. We use i in math because, although it represents something that does not exist, it actually helps (in high-level math and physics) us to calculate things that are very real indeed.

What you should know about complex and imaginary numbers is as follows:

1. They follow all the rules of regular math. You can add like terms, cancel out common factors, do the same thing to both sides, and so on. Exponent rules might apply as well; for example, $\frac{i^7}{i^6} = i$.

2. There is a pattern to powers of i .

- $i^0 = 1$ Just like everything else to the zero power is 1.
- $i^1 = i$ Just like everything else to the first power is itself.
- $i^2 = -1$ This is its definition.
- $i^3 = -i$ It's like $i^2 \times i^1$. That's the same as $-1 \times i$, which is $-i$.
- $i^4 = 1$ It's like $(i^2)^2$, which is $(-1)^2$, which is 1.

Notice that we're back to 1, so *now the pattern starts over*. In fact this pattern of four values — $i, -1, -i, 1$ — repeats infinitely for powers of i .

3. When i is in a denominator, it must be rationalized (see following page).

1. Simplify: $(4 + 3i) + (7 - 5i)$

5. What is i^7 ?

2. Simplify: $(2 + i) - (5 - 3i)$

6. What is i^{18} ?

3. Simplify: $(3 + 2i)(4 - i)$

7. What is $(i^7)(i^9)$?

4. Simplify: $(7 - 3i)(6 - i)$

8. What is $\frac{i^9}{i^4}$?

Rationalizing Denominators

[Kuta 1](#), [PurpleMath](#), [Regents](#), [MathIsFun](#)

Radicals and complex numbers (those including i) cannot stay in the denominator of a final answer. They must be *rationalized* depending on how many terms are in the denominator.

If there's just a radical or just an "i" term:

Multiply top and bottom by whatever is in the denominator, then simplify.

Ex. A: Simplify $\frac{6}{\sqrt{3}}$

Multiply top and bottom by $\sqrt{3}$:

$$\frac{6}{\sqrt{3}} \times \left(\frac{\sqrt{3}}{\sqrt{3}} \right) = \frac{6\sqrt{3}}{3}$$

Simplify:

$$2\sqrt{3}$$

Ex. B: Simplify $\frac{5}{2i}$

Multiply top and bottom by $2i$:

$$\frac{5}{2i} \times \frac{2i}{2i} = \frac{10i}{4i^2}$$

Simplify:

$$\frac{10i}{4(-1)} = \frac{10i}{-4} = -\frac{5i}{2}$$

If the denominator is a binomial:

Multiply top and bottom by the *conjugate* of the denominator: the same two terms of the binomial, but with the opposite sign in the middle.

Ex. C: Simplify $\frac{5+\sqrt{2}}{4-\sqrt{2}}$

Multiply top and bottom by $4 + \sqrt{2}$:

$$\frac{(5 + \sqrt{2})}{(4 - \sqrt{2})} \times \frac{(4 + \sqrt{2})}{(4 + \sqrt{2})}$$

Multiply:

$$\frac{(5 + \sqrt{2})(4 + \sqrt{2})}{(4 - \sqrt{2})(4 + \sqrt{2})} = \frac{20 + 9\sqrt{2} + 2}{16 - 2}$$

Simplify:

$$\frac{20 + 9\sqrt{2} + 2}{16 - 2} = \frac{22 + 9\sqrt{2}}{14}$$

Ex. D: Simplify $\frac{3i}{7+2i}$

Multiply top and bottom by $4 + \sqrt{2}$:

$$\frac{3i}{7 + 2i} \times \frac{7 - 2i}{7 - 2i}$$

Multiply:

$$\frac{3i}{7 + 2i} \times \frac{7 - 2i}{7 - 2i} = \frac{21i - 6i^2}{49 - 4i^2}$$

Simplify:

$$\frac{21i - 6i^2}{49 - 4i^2} = \frac{21i - 6(-2)}{49 - 4(-1)} = \frac{21i + 8}{53}$$

Exponential Growth and Decay

[Regents 1](#), [Regents 2](#) [MathPlanet](#), [Hotmath](#), [IXL](#)

Exponential growth or decay occurs when a quantity increases or decreases based on a repeating factor — or perhaps more simply, when it increases or decreases by a certain percentage again and again. Common examples are the growth of bacteria (the number of colonies increases by, say, 20% every hour), radioactivity (the amount of radioactive material decreases by 50% every half-life), and interest (Nora's investment grows at 6.2% per year).

The formula for exponential growth or decay is $y = C(1 + r)^t$

In this formula, y is the final value, C is the starting value, r is the rate of growth or decay, and t (which can also be x if you're graphing it) is the number of time periods.

Ex. A: Solange invests \$500 at an interest rate of 3%. How much money will she have 5 years from now?

$$y = 500 \left(1 + \frac{3}{100} \right)^5$$

$$y \approx \$579.64$$

Ex. B: The half-life of a certain radioactive material is 11 days. Write an equation to represent after how many days 21 grams will be left from an initial sample of 74 grams?

$$21 = 74(0.5)^x$$

1. Jolene's investment has done poorly, losing 4% per year. How much remains 3 years after her initial investment of \$3,000?

3. A colony of bacteria increases its size by 1.5% per minute. Write an equation representing the number of hours until the size doubles from its current size.

2. Only 7% of a sample of radioactive material remains after 15 days. Write an equation that could be used to determine the half-life of this material.

4. Mr. Peabody's YouTube channel gains 8% more subscribers every week. If he currently has 800 subscribers, how many subscribers will he have in 10 days?

Radical Equations[PurpleMath](#), [MathIsFun](#), [Kuta](#)

Solve radical equations by isolating the radical and squaring both sides of the equation. You must check potential solutions to radical equations to confirm that they work.

Ex. A: $\sqrt{x + 20} - x = 0$

Isolate the radical:

$$\sqrt{x + 20} = x$$

Square both sides:

$$x + 20 = x^2$$

Rearrange, factor, and solve:

$$\begin{aligned}x^2 - x - 20 &= 0 \\(x - 5)(x + 4) &= 0 \\x = 5 \text{ and } x = -4\end{aligned}$$

Check $x = 5$:

$$\begin{aligned}\sqrt{x + 20} - x &= 0 \\ \sqrt{5 + 20} - 5 &= 0 \\ 0 &= 0 \\ x = 5 \text{ is a solution}\end{aligned}$$

Check $x = -4$:

$$\begin{aligned}\sqrt{-4 + 20} - (-4) &= 0 \\ \sqrt{16} + 4 &= 0 \\ 8 &\neq 0 \\ x = -4 \text{ is NOT a solution}\end{aligned}$$

Ex. B: $\sqrt{x - 3} = x - 5$

Square both sides:

$$x - 3 = (x - 5)^2$$

Expand:

$$x - 3 = x^2 - 10x + 25$$

Rearrange, factor, and solve:

$$\begin{aligned}x^2 - 11x + 28 &= 0 \\(x - 4)(x - 7) &= 0 \\x = 7 \text{ and } x = 4\end{aligned}$$

Check $x = 4$:

$$\begin{aligned}\sqrt{4 - 3} &= 4 - 5 \\ \sqrt{1} &= -1 \\ 1 &\neq -1 \\ x = 4 \text{ is NOT a solution}\end{aligned}$$

Check $x = 7$:

$$\begin{aligned}\sqrt{7 - 3} &= 7 - 5 \\ \sqrt{4} &= 2 \\ 2 &= 2 \\ x = 7 \text{ is a solution}\end{aligned}$$

1. $x - 1 = \sqrt{5x - 9}$

2. $\sqrt{3x - 5} = x - 3$

Evaluating the Discriminant of a Quadratic Function

[PurpleMath](#), [MathIsFun](#), [Kuta](#)

In the quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the portion under the radical, $b^2 - 4ac$, is called the *discriminant*. The value of the discriminant determines the number and type of solutions to a quadratic equation as follows:

Value of $b^2 - 4ac$	Solutions: number and type	Graph Behavior
> 0	2 real solutions	2 x -intercepts
$= 0$	1 real solution (a double root)	Touches x -axis at 1 point
< 0	0 real solutions; 2 complex ($a + bi$) solutions	No x -intercepts

1. Describe the solutions and graph of $y = x^2 - 7x + 3$.

4. What is c if the equation $y = x^2 + 5x + c$ has exactly one solution?

2. Describe the solutions and graph of $y = 2x^2 - 4x + 3$.

5. What is the minimum possible integer value of a given that the function $f(x) = ax^2 + 5x + 3$ does not have any real zeroes?

3. Describe the solutions and graph of $y = x^2 + 8x + 16$.

6. What is the largest possible integer value of c given that the graph of the function $g(x) = 2x^2 - 7x + c$ has two distinct x -intercepts?

Tolerance Questions

[BrainGenie](#)

Tolerance questions involve backtracking from a graph (or the information necessary to create one) to an absolute value inequality. Consider the following example:

Determine an absolute value expression indicating that the temperature, t , at which ice cream is to be held must be no less than 28 nor greater than 32.

There are two ways to express this information. One way is that the temperature must be between 28 and 32. The other is that the temperature should be 30, with a tolerance of ± 2 degrees.

In this second way, we can say that the difference between t and 30 must be less than or equal to 2 degrees. This allows us to construct an absolute value inequality:

$$|t - 30| \leq 2 \quad \text{or} \quad |30 - t| \leq 2$$

Because of the absolute value signs, these two expressions are exactly equivalent.

1. Create an absolute value expression indicating that the new manager of the Products Division will make a salary, y , that is not less than \$55,000 or more than \$75,000.

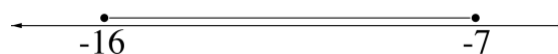
2. A “mercy rule” in a baseball league specifies that a game is over if one team has at least 10 runs more than another. For two teams x and y , create an absolute value expression indicating when the game is over.

3. Create an absolute value inequality indicating that a number x must be between 0.004 and 0.04.

4. Create an absolute value inequality indicating height requirement, h , in inches for a circus performer who must be either taller than 7’6 or shorter than 3’10.

Create absolute value expressions representing the graphs shown:

5.



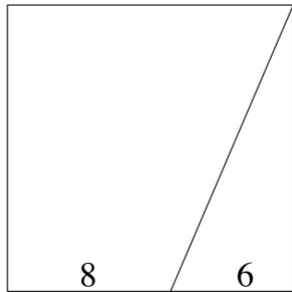
6.



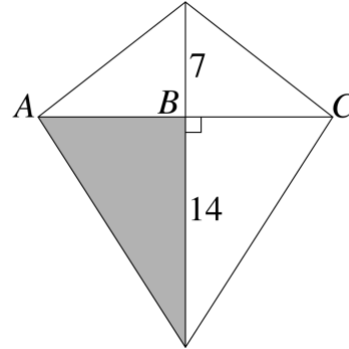
Geometric Probability

[Algebra-Class](#), [Virtual Nerd](#), [OML](#)

Geometric Probability is calculated as $\frac{\text{Desired Area}}{\text{Total Area}}$.



1. What is the probability that a point chosen at random in the square above will also be in the triangle?

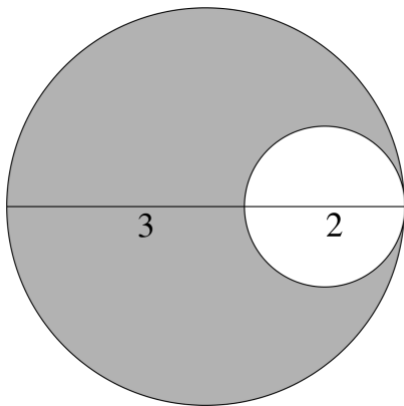


2. What is the probability that point chosen at random in the diamond above will also be in the shaded area?

Using Scale Factors

There is also a very cool shortcut that can be used to solve many geometric probability questions. This shortcut is based on the fact that units of length correspond to an exponent of 1 (for example, inches); units of area to an exponent of 2 (inches squared); and units of volume to an exponent of 3 (units cubed).

The shortcut is as follows: *The one-dimensional scale factor between two objects can be squared to find their scale factor in two dimensions and cubed to find their scale factor in three dimensions.* This shortcut also works in reverse using square roots, cube roots, etc.



What is the probability that a point chosen at random in the large circle above will be in the shaded area?

Solution:

The scale factor of the circles in one dimension is $\frac{2}{3}$.

Squaring the one-dimensional scale factor gives us their scale factor in two dimensions: $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$. This means that the ratio of their areas is 4:9.

If the relative areas of the smaller and larger circles are 9 and 4, then the shaded area is $9 - 4 = 5$.

The probability is $\frac{\text{Desired Area}}{\text{Total Area}} = \frac{5}{9}$.

Geometric Probability Practice

3. An equilateral triangle is inscribed in a circle. If a point is selected at random inside the circle, what is the probability that the point will also be in the triangle?

4. A square is circumscribed around a circle. What is the probability that a randomly selected point in the square is also in the circle?

5. The side of one equilateral triangle is three times as long as the side of another equilateral triangle. What is the ratio of their areas?

6. A rectangle with sides in a ratio of 3:1 is fully contained inside another rectangle whose sides are in the same ratio but each of whose legs is 1.5 times as long. What is the probability that a point in the large rectangle is also in the small rectangle?

7. A scale model of a statue requires sixteen times as much sealant to cover its surface. What is the ratio of the height of the scale model to the height of the statue?

8. What is the ratio of those statues' volumes?

9. What percent of the volume of a cube with sides 2.5 units long is the volume of a cube with sides 1 unit long?

10. The ratio of the volumes of two spheres is 64:125. What is the ratio of their radii?

Both/Neither Formula

$A + B - (\text{Items with both characteristics}) + (\text{Items with neither characteristic}) = \text{Total Items}$

1. In a group of 30 writers, 13 can write with their left hand and 24 can do so with their right hand. How many are ambidextrous?

2. In a neighborhood of 80 houses, 32 have a fireplace, 43 have a pool, and 11 have both. How many have neither?

3. At Quicksand High School, there are exactly 90 juniors, each of whom does at least one activity out of photography, chorus, and art. 48 students do photography and 20 students do art. What is the minimum number of students who do chorus?

4. In a group of 75 students, each student lives with at least one out of a sister, brother or grandparent. 33 students live with a sister and 19 live with a brother. What is the maximum number of students who could live with a brother and a grandparent?

5. At Jones High School, there are exactly 122 seniors, each of whom does at least one activity out of either a sport, a club, or student council. 93 students do a sport and 13 do student council. If exactly 48 students participate in a club and at least one other activity, what is the maximum number of students who can be in a club?

- (A) 61 (B) 74 (C) 77
(D) 80 (E) 106

Shared Work Formula

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{t}$$

Where x = one time to complete a task, y = another time time, etc., and t = combined time.

6. One machine can clean the floor of a certain room in 60 minutes, while a second machine can do so in 30 minutes. If the two machines work at the same time, how long will it take to clean the floor?

7. Jane can paint a wall in 40 minutes and Joe can paint it in 20 minutes. If Joe works for 5 minutes before Jane joins him, how much longer will it take to finish the wall?

8. Joe can mow a lawn in 4 hours, and Jane can mow it in 3 hours. How long would it take them together to mow the lawn?

9. Bobby can do his family's yard work in 1 hour. If he and his brother Billy work together, they can complete the work in 24 minutes. How long would it take Billy to complete the work if he did it alone?

Counting

$$\text{Counting} = \text{Distance} + 1$$

10. How many integers are there between 1 and 3, inclusive?

13. How many even integers are there between 2 and 42, inclusive?

11. How many integers are there between 37 and 51, inclusive?

14. A teacher assigns math homework starting at question #27 and doing only odd-numbered questions. If the teacher wants her students to do exactly 35 questions, what is the highest-number question they should complete?

12. Phil is selecting a group of prizewinners in a contest. He selects entrant number 250 and every 150th entrant after this as a winner. If Phil selects 40 prizewinners, what is the entrant number of the last person selected as a winner?

15. How many multiples of 7 are there on a list of numbers which starts with 91 and ends with 329?

Logarithms

[MathIsFun](#), [PurpleMath](#), [Kuta](#)

$\log_a b = x$ means that $a^x = b$. For example, $\log_2 32 = x$ means that $2^x = 32$, so $x = 5$.

Solve for x :

1. $\log_7 x = 2$

3. $\log_x 1728 = 3$

2. $\log_3 243 = x$

4. $\log_x \frac{81}{256} = 4$

Answers**Unit 1****Transitions**

1. B, and
2. A, though
3. C, and besides
4. B, but
5. C, although
6. A, in other words
7. B, essentially
8. C, more generally
9. A, for example
10. C, based on
11. A, instead

Punctuation Post-Test

1. C
2. A
3. A
4. B
5. B
6. B
7. C
8. C

Smart Math Strategy: Guess-and-Check

1. C
2. A
3. E
4. B
5. D
6. D
7. E
8. E
9. B
10. A
11. A
12. E
13. B
14. C
15. E
16. C

Guess-and-Check Post-Test

1. A) 7°

2. C) 16
3. C) \$10.00
4. D) 30

UNIT 2**Pronouns - Clear and Proper Reference**

(Subject, Pronoun, Correct - yes or no)

1. Unclear subject, he, no (should be "they" or indicate which person said hello)
2. stores, their, no (televisions are declining in value, not stores)
3. city, they, no ("they" refers to an unmentioned group of people, not to "city")
4. Unclear, it, no (could refer correctly to comprehension or incorrectly to projects)
5. Unclear, their, no (could refer to the advisor's friends or the partner's, but either way it should be "his" or "her")
6. Unclear — who's they?
7. Someone, they, no (should be "he" or "she")
8. posters, them, yes
9. city, its, yes
10. team, her, no (should be "their")
11. class, their, no (should be "its")

Pronouns Post-Test

1. A
2. C
3. A
4. C
5. B
6. B
7. C
8. C

Subject/Verb Agreement

1. cat/correct
2. machines (should be "are")
3. lady/correct
4. chicken (should be "makes")
5. parks (should be "are")

6. answer/correct
7. moment (should be "is")
8. Folks/correct
9. owner (should be "comments")
10. guy/correct
11. Surfers (should be "remind")
12. one (should be "is"; the "not" is irrelevant)
13. one/correct
14. they/correct
15. Nobody (should be "was")
16. car (should be "is")
17. Alonzo/correct
18. group (should be "is")

More with Verbs

1. idea (agrees with "is")
2. One/is
3. piles/were
4. one/sells
5. articles/relate
6. fact/carries
7. had (matches with came)
8. was (showed)
9. had been carrying (realized)
10. remains (now)
11. Having lived (because he's back now)
12. will rest (future, opportunities)

Verbs Post-Test

1. A
2. C
3. A
4. C
5. A
6. A
7. A
8. C

Parallel Structure

1. ...and great coaching staff (to match "its").
2. ...and stare (to match "look").
3. ...did mind being (to match "doing").
4. ...somehow sturdy (to match "delicate").

5. Correct: it was brimming with passion; it was grounded in reason.
6. Correct: fear is grammatically equivalent to desire.

Parallel Structure Post-Test

1. A
2. C
3. C
4. B
5. B
6. A

Smart Math Strategy: Making Up Numbers

1. B
2. E
3. C
4. B
5. E
6. C
7. E
8. D
9. E
10. B

Making Up Numbers Post-Test

1. D
2. E
3. C
4. A

Systems of Equations

1. $(-3, 4)$
2. $(-1, 8)$
3. $(7, -2)$
4. $(7, 6)$

Linear Eq's/Ineq's Post-Test

1. $x = 2$
2. All real numbers
3. $x > 5$
4. I and II
5. $14 = 9 + 1.25(x - 1)$; 5 toppings
6. $d = 8\frac{2}{3} - \frac{3}{4}h$

- Hot dogs \$4, Sodas \$2.50
- $600 + 0.5(c + 3) = 2c$; 401 cups

Rates

- \$300/month
- \$300/month
- \$14,000
- $v(m) = 14,000 + 300m$
- August 2015
- \$20,300
- See #4 above

Ratios and Proportions

- 270 words
- 3 machines
- \$2,880
- 99 games
- 25 chocolate chip cookies
- E
- $\frac{20}{21}$
- \$1,155
- 75 good profiles
- $2\frac{2}{5}$ cups of sugar

Ratios and Proportions Post-Test

- 250 calories
- 60 machines
- 1,080 athletes
- 16 plays

Statistics: Measures of Central Tendency and Spread

- mean 1.125 days, median 1 day, mode 0 days, range 6 days
- means are equal (83), Jimmy's median is higher (83 to 80), Jimmy has two modes (83 and 84), Deanna has one mode (95), Deanna has a larger range (22 to 5), and Deanna has a larger standard deviation.

Statistics Post-Test

- Mean
- Mean: $1\frac{2}{9}$, Median: 1

- Gary had a higher mean and median. Only Michael had a mode (233). Michael has a higher range and higher standard deviation.
- Gary, because he has a lower standard deviation.
- Above 225, Michael. Above 200, Gary. Better...depends.

Coordinate Geometry, Part 1

- 2
- 6
- $m = \frac{3}{5}$, $b = -3$
- $Slope = \frac{1}{6}$, $intercept = \frac{7}{6}$
- $2x - 3y = 6$
- $y = \frac{3}{7}x + 2$

Coordinate Geometry Post-Test

- y -int. 8, $m = -\frac{1}{10}$
- y -int. is starting depth
- slope is rate of change of depth
- $y = -\frac{1}{10}x + 8$
- $23\frac{1}{3}$ minutes
- 15
- $-\frac{2}{3}$
- $E = 3500 + 300x$
- \$3,500
- \$300

Logic and Process of Elimination

- C (about 25% more than 7,500: 7,703 is way too low, but 25% more than 8,000 is 10,000, so the answer must be lower.)
- C (With all those older people, Zach must be pretty young.)
- A (Just look at it: the diagram looks reasonable, and x isn't anything like 70 or 80.)
- B (Has to be significantly less than 30% if it's three times in a row, but it's not as low as $1/400$.)

Reading ACT Figures and Tables

1. 19 (± 1) species
2. There is no significance. The graphs are of different units and on different scales. Nothing actually meets at that point.
3. 10 (± 1) species
4. D

UNIT 3**Descriptive Clauses**

1. "Julie found a dress that was"; change so that "Julie" is the first word after the modifier.
2. Correct
3. "while looking"; change so that "I" and not "care keys" is doing the looking.
4. "Freddie found himself in a dilemma during the election"
5. "The quarterback of the Tigers,,"; change so that the Tigers, not the quarterback, are one of the best teams.
6. "to the Farmers' Market on Saturday"; change so that Saturday, and not the Farmers' Market, is a day with little work.
7. "The tomato"; the tomato, not its categorization, is a fruit.
8. "John let the barn sit for years"; John, not the barn, was one to procrastinate.
9. Correct.
10. "the footprints of the astronauts who visited the moon"; the footprints are perfectly preserved, not the astronauts.

Subjects and Objects Practice

1. whom (the subject "manager" immediately follows), correct.
2. us (Alejandra is the subject, Sophia and "us" are both objects).
3. Correct.
4. them ("decisions" is the subject, made by

"them", the object).

5. Correct.
6. her ("towards" makes "her" an object), correct.
7. Correct.
8. we (forget customers: "we are always right"), correct.
9. Correct (the subject "we" immediately follows).
10. I ("It was lucky that I made that decision)

Subordinate Conjunctions

1. Incorrect; the two parts of the sentence make sense together, so "Even though" is inappropriate.
2. Correct.
3. Incorrect; the two parts of the sentence make sense together, so "but" is inappropriate.
4. Correct.
5. The two parts of the sentence express opposite ideas, so "along with" is not appropriate.
6. Incorrect; "Given that" is inappropriate because the second part of the sentence does not result from the first.

However, etc.

1. but (conjunction)
2. Though, Although (conjunction)
3. however, though (adverb)
4. but (conjunction)
5. However (adverb)
6. however, though (adverb)
7. but (conjunction)
8. Although, Though (conjunction)
9. though, however (adverb)
10. however (adverb)
11. but (simple contrast)
12. though, although (conjunction)
13. however, though (adverb)
14. but (conjunction)
15. Though, although (conjunction)

Run-Ons

1. No (all sentence parts are joined

- correctly).
2. Yes (two sentences with just a comma between).
 3. Yes (two sentences with just a comma between).
 4. No (correct use of semicolon between complete sentences).

Fragments

5. Sentence; has subject and verb.
 6. Fragment; the subject "answer" has no verb.
 7. Sentence; has subject and verb.
 8. Fragment; all modifiers with no subject or verb.
- * Words like of, that, than, which, and w-words can turn sentences into fragments

Coordinate Geometry, Part 2

1. (1, 2)
2. (12, 17)
3. 10
4. -3
5. 10 miles
6. 25
7. a. 4, b. $-\frac{1}{4}$
8. a. $-\frac{1}{3}$, b. 3
9. 3
10. -1

Percentages and Percent Change

1. \$12,000
2. 80%
3. 80 ounces
4. 28%
5. B
6. \$60
7. 0.083
8. \$20
9. \$48
10. 8.5%
11. 175%
12. -1%
13. 80%
14. \$1,450

15. 15%
16. 15%
17. 60 inches
18. 650%
19. 20%

Percentages Post-Test

1. \$16,600
2. \$740
3. 4,875
4. 27.1%

Unit Conversion/Dimensional Analysis

1. \$2.22/yard
2. 396 calories/pound
3. 6.34 days/year
4. 20 square yards/dozen people
5. 13.8 kg/cubic foot
6. 22.75 inches

Rates and Dimensional Analysis Post-Test

1. 22,175
2. $43\frac{8}{9}$ mps
3. ≈ 64.53 ft/sec
4. $\frac{2}{9}$ oz/in³

Two-Way Tables

1. $\frac{27}{40}$; $\frac{41}{60}$
2. No; boys and girls prefer dogs to cats in almost exactly equal proportions.
3. $\frac{17}{42}$
4. $\frac{46}{71}$
5. Yes; 40% of light-haired students had light eyes, but only 29% of dark-haired students had light eyes.
6. $\frac{22}{94}$, reduce to $\frac{11}{47}$
7. No; the same *number* of people prefer water, but it equates to 11% of junk food people and only 7% of upscale food people.
8. $\frac{22+7}{68+14} = \frac{29}{82}$

9. Yes; of the junk food people, 70% preferred soda, while of the upscale food people, only 23% preferred soda.

10. ≈ 11 people. $\frac{65}{94} \approx \frac{11}{16}$

Solving Linear Equations/Inequalities in Terms of Other Variables

1. $C = \frac{5}{9}(F - 32)$

2. $r = \sqrt[3]{\frac{3}{4\pi}V}$

3. $v_i = v_f - at$

4. $x = -\frac{3}{r-4}$ or $x = \frac{3}{4-r}$

More About 2-by-2 Systems of Linear Equations

1. -8.5

2. 6.5

3. $-\frac{4}{3}$

4. 10

Completing the Square

1. $y = (x + 6)^2 - 5$

2. $(x + 5)^2 + (y - 2)^2 = 5^2$

Factoring by Grouping

1. $-4, -1,$ and 1

2. $-5, \frac{1}{3},$ and 1

Right Triangles

1. 13

2. 48

3. 4 and $\sqrt{114}$

4. 24 inches

5. 15 inches

6. 240 miles

7. $5\sqrt{2}$ mm

8. $3\sqrt{2}$ in

9. 6 feet and $6\sqrt{3}$ feet

10. $2\sqrt{3}$ cm and $4\sqrt{3}$ cm

11. 12

12. $8\sqrt{3}$

13. $5\sqrt{3}$

14. 13

Circles

1. Diameter 12 , Circumference 12π , Area 36π

2. Radius $2\sqrt{3}$, Circumference $4\pi\sqrt{3}$, Area 12π

3. Radius 4 , Diameter 8 , Area 16π

4. Radius $4\sqrt{3}$, Diameter $8\sqrt{3}$, Circumference $8\pi\sqrt{3}$

Working With Exponents and Roots

1. $\frac{x^2y^2}{z^6}$

2. $\frac{x^2y^9}{z^8}$

3. $\frac{y^7}{x^3z^3}$

4. $\frac{a^{12}}{b^{15}}$

5. 10

6. 7

7. 11

8. 4.1

9. \sqrt{ab}

10. $a\sqrt{b}$

11. 7

12. 6

13. $4\sqrt{2}$

14. $3\sqrt{6}$

15. $3\sqrt{5}$

16. 2

17. $x^5\sqrt{x}$

18. $5a\sqrt{2a}$

19. $3x^4\sqrt{2x}$

Functions

1. -4

2. $4x^2 - 6x$

3. $x = -3$ or -11

4. $k = 3$

5. 46

6. $x = 4$ and $x = -1$

7. 2

8. A

9. a. $4x^2 + 4x + 1$, b. $2x^2 + 1$

10. $30\frac{1}{3}$
11. -5
12. $3 < x < 8$
13. $\frac{1}{3}$
14. $m = 3$
15. $\frac{4}{4x-1}$

Geometry Practice Questions

Example question: 18π

1. D
2. 17.0 (exact answer is $12\sqrt{2}$)
3. 13.5
4. 24
5. 20
6. 100°

Probability Distributions

1. 0.126
2. 55.3%
3. 2.81
4. 2.81
5. Same: "expected value" means "average"
6. ~ 1.94 special diets per child

UNIT 4

Adverbs and Adjectives

Directions: correctly, necessary

1. quickly, urgent
2. informative
3. starkly, seemingly
4. interesting, famous

Comparisons

Correct and Equivalent Items

1. Incorrect; compares haircut to hair.
Should be haircut to haircut.
2. Incorrect; compares green car to Ellen.
Should be, "Butch liked the green car much more than Ellen liked the green car," or "much more than Ellen did."
3. Incorrect; compares a star to the light from a star; should compare light to light or star to star.

4. Correct; compares differences to differences.
5. Incorrect; compares the cost to a model. Should compare cost to cost or model to model.

Proper Word Choice

6. Incorrect; should be "more" because there are two items.
7. Incorrect; should be "number" because oranges can be counted.
8. Incorrect; should be "greater" because only a quantity can be higher, and respect cannot be counted.
9. Incorrect; should be "many more" because arguments can be counted.
10. Incorrect; wealth cannot be counted, so "amount" or "frequency" should be used instead of "number".

Idiomatic Prepositions

1. From
2. upon
3. "in search of an answer" or "to search for an answer"
4. in
5. to
6. from or since

Multiplying Quadratics and Other Polynomials

1. $x^2 + 3x$
2. $x^2 + 10x + 24$
3. $x^2 + 6x + 9$
4. $3x^3 - 9x^2 - 30x$
5. $6x^2 - x - 2$
6. $x^2 - 9x + 8$
7. $x^2 - 16$
8. $8x^4 - 2x^2$

Solving Quadratics and Other Polynomials

1. $x = 0$ and 2
2. $x = -7$ and -2
3. $x = -2$
4. $x = -\frac{3}{2}$ and 4

5. $x = -2, 2, -i$, and i
6. $x = -5, 0$, and 5
7. $x = 0$ and $\frac{5 \pm 3\sqrt{5}}{2}$
8. 3
9. 2.5

Probability Practice

1. $1/32$
2. $5/33$
3. $1/125$
4. $64/125$
5. $12/25$
6. $1/21$
7. $10/21$
8. $8/125$
9. $1/5$
10. $7/13$

Permutations and Combinations

1. ${}_6P_4 = 360$
2. ${}_7C_4 = 35$
3. ${}_6C_4 = 15$
4. ${}_{17}C_2 = 136$
5. ${}_7P_5 = 2520$
6. ${}_8P_4 = 1680$
7. 60 (one-of-each, not a P or C)
8. ${}_8P_3 = 336$
9. ${}_6C_3 = 20$

Triangles in General

1. 12
2. 4
3. $\frac{9\sqrt{3}}{2}$
4. 120
5. 96°
6. Shortest: 3, Longest: 17
7. B
8. $9\sqrt{3}$
9. 48
10. $12\sqrt{2}$
11. 100°
12. C ($a + b > c$, not = to)
13. 10.5

Rectangles

1. 187
2. 77
3. $\frac{4}{9}$

Squares

4. $20\sqrt{2}$
5. 12.25
6. 49
7. D
8. 225
9. 16

Other Quadrilaterals

Trapezoids

1. 55
2. 400

Parallelograms

3. $48\sqrt{2}$
4. 10
5. 130
6. 63
7. II only

Averages

1. 12
2. $3x + 2$
3. $\frac{3}{2}$
4. \$23
5. $a = -3$
6. $x + 12$

Complex Averages

1. 66 mph
2. 250 pounds
3. 0.05
4. \$64,500
5. 4 students
6. 76%
7. 3 games
8. $7\frac{2}{3}$ sales per month
9. 74%
10. C

Sectors of Circles

- $\frac{4\pi}{3}$
- 120°
- 72°
- $\frac{\pi}{2}$
- 2π
- 18.75π cm

UNIT 5**Radians**

- $\frac{\pi}{4}$
- 30°
- $\frac{7\pi}{6}$
- 300°

Absolute Value

- $x = 10$ or $x = -2$
- $x = 3$ or $x = -4$
- $x = -1$ or $x = -13$
- $x = 4$ or $x = 2$
- $x = -2$ or $x = -2/3$
- No Solution; absolute value would have to be negative, which is impossible.
- B (only 1; the rest have 2)
- E; absolute value can't be negative
- E

Absolute Value Inequalities

- $1.5 < x < 4.5$
- $-\frac{5}{2} \leq x \leq \frac{7}{2}$
- $x < -15.5$ or $x > 1.5$
- $x \leq -21$ or $x \geq 13$
- E
- $x \leq -6$ or $x \geq 9$
- E
- A

Sequences

- 23
- 19
- Day #64 (not 63!)
- 162
- 40.5

- $\frac{1}{9}$
- 2
- 122 (work back from 365!)
- 45 (times 3, add 3, times 3, add 3...)

Repeating Sequences

- Thursday
- B
- 3
- D
- L
- D
- 8
- 1
- A

Geometric Solids

- Volume = 40 ft^3 , Surface Area = 76 ft^2 , Diagonal = $3\sqrt{5} \text{ ft}$
- Cannot be determined!
- E
- Side Length = $\sqrt{3} \text{ in}$, Surface Area = 18 in^2 , Diagonal = 3 in
- 30% $\left[\text{calculate as } \left(\frac{2}{3}\right)^3 \right]$
- 27 cm^3
- B

Relationships Between Variables

- Linear, Indirect, Strong, $r \approx .8$
- Exponential, Indirect, Weak, $r \approx .4$
- Exponential, Direct, Strong, $r \approx .8$
- No relationship, $r \approx 0$
- Linear, Indirect, Weak, $r \approx .2$
- Linear, Direct, Strong, $r \approx .9$

Circles: Equations

- Circle centered at $(2, -1)$ with radius 2.5.
- $(x + 1)^2 + (y - 1)^2 = 4$
- Circle centered at $(0, 1.5)$ with radius 2.
- $x^2 + y^2 = 9$
- Circle centered at $(1.5, -1.5)$ with radius 1.5 (left and top of circle are

tangent to axes).

6. $(x + 5)^2 + (y + 2)^2 = 1$

Word Problem Practice Set

- 9
- $x = -2, y = 5$
- Alex is 2, Billy 6, Charlie is 12
- 4
- $\frac{7}{8}$
- 48 eggs
- 12
- Any product of 490 and a prime number; ex. 980, 1470, 2450. 490 is not an answer because 1 is not prime.
- 7
- 216

More Trigonometry

- $\csc^2 x$
- $\sin \theta = \frac{5}{13}, \cos \theta = \frac{12}{13}, \tan \theta = \frac{5}{12}, \csc \theta = \frac{13}{5}, \sec \theta = \frac{13}{12}, \cot \theta = \frac{12}{5}. m\angle \theta \approx 22.62^\circ$ and $\approx .395$ rad
- 11.2
- 0.8
- Angles are .22 radians and 1.35 radians. Sides are 1.43 and 6.34.
- 67°
- $\frac{2\sqrt{5}}{5}$

Direct/Inverse Variation/Proportion

- 16
- 10
- C
- 3
- 18
- E
- I and III
- 45,000
- I and II
- 6
- 15
- III

Complex Numbers

- $11 - 2i$
- $-3 + 4i$
- $14 + 5i$
- $39 - 25i$
- $-i$
- 1
- 1
- i

Exponential Growth and Decay

- \$2654.21
- $0.07 = (0.5)^{\frac{15}{x}}$
- $2 = (1.015)^{60h}$
- 893

Radical Equations

- 2 and 5
- 7 (2 does not work)

Evaluating the Discriminant of a Quadratic Function

- 2 real solutions. The graph intersects the x -axis twice and opens up.
- 0 real solutions, 2 complex solutions. The graph does not intersect the x -axis.
- 1 real solution. The graph is tangent to the x -axis at one point.
- 6.25
- 3
- 6

Tolerance Questions

- $|65,000 - y| \leq 10,000$ or $|y - 65,000| \leq 10,000$
- $|x - y| \geq 10$ or $|y - x| \geq 10$
- $|x - 0.022| \leq 0.018$ or $|0.022 - x| \leq 0.018$
- $|h - 68| > 22$ or $|68 - h| > 22$
- $|x + 11.5| \leq 4.5$ or $|11.5 - x| \leq 4.5$
- $|x - 37| \geq 34$ or $|37 - x| \geq 34$

Geometric Probability

- $\frac{3}{14}$

2. $\frac{1}{3}$
3. $\frac{3\sqrt{3}}{4\pi}$
4. $\frac{\pi}{4}$
5. 9:1
6. $\frac{4}{9}$
7. 4:1
8. 64:1
9. 6.4%
10. 4:5

Both/Neither Formula

1. 7
2. 16
3. 22
4. 19
5. C

Shared Work Formula

6. 20 minutes
7. 10 minutes
8. $\frac{12}{7}$ hours
9. 40 minutes

Counting

10. 3
11. 15
12. 6100
13. 21
14. 95
15. 35

Logarithms

1. 49
2. 5
3. 12
4. $\frac{3}{4}$